1. (10 marks) (280 marks remain)

If $x = \frac{111110}{111111}$, $y = \frac{222221}{222223}$, and $z = \frac{333331}{333334}$, place x, y, and z in order from largest to smallest.

Observe that

$$x = 1 - \frac{1}{111111} = 1 - \frac{6}{666666},$$

and

$$y = 1 - \frac{2}{222223} = 1 - \frac{6}{666669},$$

and

$$z = 1 - \frac{3}{333334} = 1 - \frac{6}{666668}.$$

It is clear that

$$\frac{1}{666669} < \frac{1}{666668} < \frac{1}{666666},$$

hence $y > z > x$.

2. (10 marks) (270 marks remain)

Let x_n be the nth odd composite number. What is x_{x_1}?

Note: A number is composite if it is not prime, i.e. it has greater than two factors. The number 1, however, is neither composite nor prime.

The first ten odd composite numbers are $9, 15, 21, 25, 27, 33, 35, 39, 45, 49$.

Hence $x_1 = 9$, and $x_{x_1} = x_9 = 45$.

3. (10 marks) (260 marks remain)

During the refreshments served after the MUMS seminar last Friday, Jeremy ate Tim Tams for 1 minute whilst Sam ate for 9 minutes. The total number of Tim Tams consumed by the pair was 8. If, after the next seminar, they consume 11 due to Jeremy eating for 2 minutes whilst Sam eats for 3 minutes, what is Jeremy's rate of consumption in Tim Tams per minute? (Assume that they each eat at the same rate after both seminars.)

Let x be Jeremy's rate of consumption in Tim Tams per minute, and let y be Sam's rate of consumption in Tim Tams per minute.

Hence

$$x + 9y = 8, \tag{1}$$

and

$$2x + 3y = 11. \tag{2}$$

From (2), we obtain $3y = 11 - 2x$, and substituting into (1) yields

$$\begin{aligned} x + 3(11 - 2x) &= 8 \\ -5x &= -25 \\ x &= 5. \end{aligned}$$

Hence Jeremy's rate of consumption is 5 Tim Tams per minute.

4. (10 marks) (250 marks remain)

Norm and Dan are in a beauty contest for which the second prize is \$10. Unable to decide, the judges devise a (rather silly) random experiment to determine the winner. They have two fair coins and two French coins that yield *face* with 60% probability and *pile* with 40% probability. All four coins are tossed. Norm is declared the winner if there are fewer than two heads and fewer than two *face*s. What is the percentage probability of Dan winning?

The probability of getting two heads is

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4},$$

and the probability of getting two *face*s is

$$\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}.$$

Thus, the probability of getting fewer than two heads (i.e. *not* getting two heads) is

$$1 - \frac{1}{4} = \frac{3}{4},$$

and of getting fewer than two *face*s is

$$1 - \frac{9}{25} = \frac{16}{25}.$$

Hence the probability of Norm winning is

$$\frac{3}{4} \times \frac{16}{25} = \frac{48}{100},$$

so the probability of Dan winning is

$$1 - \frac{48}{100} = \frac{52}{100}.$$

Therefore the percentage probability of Dan winning is 52.

5. (10 marks) **(Change runner now)** (240 marks remain)

Given that

$$2\sqrt{3\left(\frac{2}{4}\cdot\frac{3}{5}\cdot\frac{4}{6}\cdot\cdots\cdot\frac{x}{y}\right)} = 1,$$

what is $x+y$?

Observe that according to the pattern, $x = y-2$. Thus,

$$\frac{2}{4}\cdot\frac{3}{5}\cdot\frac{4}{6}\cdot\cdots\cdot\frac{y-2}{y} = \frac{2\cdot 3}{(y-1)y}.$$

Therefore,

$$\begin{aligned} 2\sqrt{3\left(\frac{2\cdot 3}{(y-1)y}\right)} &= 1 \\ \frac{2\cdot 3}{(y-1)y} &= \frac{1}{12} \\ (y-1)y &= 72 \\ y^2 - y - 72 &= 0 \\ (y-9)(y+8) &= 0. \end{aligned}$$

Since $y > 0$, it follows that $y = 9$. Hence $x = 7$ and $x + y = 7 + 9 = 16$.

6. (10 marks) (230 marks remain)

Suppose that $x + x^{-1} = 5$. What is $x^3 + x^{-3}$?

Notice that

$$(x + x^{-1})^3 = x^3 + 3x + 3x^{-1} + x^{-3}.$$

Therefore,

$$\begin{aligned} x^3 + x^{-3} &= (x + x^{-1})^3 - 3(x + x^{-1}) \\ &= 5^3 - 3 \times 5 \\ &= 125 - 15 \\ &= 110. \end{aligned}$$