Worked solutions are provided for the first 15 questions, only hints and answers will be given for the other 10.

1. To start off, let's call the 3 Litre bucket A, and the 5 Litre bucket B.

   Fill B completely, and pour 3L from B to A. In doing so, we've filled A up, once.

   Empty A, and and pour the remaining 2L of water within B to A. B is now empty, A has 2L in it, and still only once have we filled A up to the brim.

   Fill B completely, and pour enough water from B to A, so as to completely fill A up. We now have 4L of water in B and we've only twice have we filled A up completely.

   Ans: 2 (Or twice)

2. Every second, Thara covers 9m in one direction and Yi covers 6m in the other: together, they cover 16m per second. This means that the two of them will cover the whole oval in $\frac{400}{16} = 25$ seconds. Therefore, Thara covered $25 \times 9 = 225$ metres.

   Ans: 225m

3. Let the respective ages of Yi, Alisa and Han be y,a and h. From the question, we can set up the following algebraic equations;

   $a = y + 1$

   $h = a + 2$

   $y + a + h = 118$

   Solving the simultaneous equations shows that $a = 39$.

   Ans: 39

4. The answer can't be five, because Elmo and Andrew would be telling the truth.

   It can't be four, because Daisy and Andrew would be telling the truth.

   It can be three, with Daisy and Andrew telling the truth.

   It can't be two, because three people (Elmo, Daisy and Bertas) would be lying.

   It can't be one, because three people (Elmo, Daisy and the Chipmunk) would be lying.

   It can't be zero, because what Bertas, the Chipmunk, Daisy and Elmo say are in conflict.

   Ans: 3

5. Due to the pairing of the numbers, the four numbers of any page will sum up to $1 + 2 + 399 + 400 = 802$. (Prove this yourself, I can't be bothered.) Therefore, the sum of the other 3 pages will be $802 - 67 = 735$

   Ans: 735

6. It takes $\frac{2}{v}$ minutes to fill up the first half of the 4L tank, and a further $\frac{2}{v-2}$ minutes to fill up the rest. Therefore, we have: $\frac{2}{v} + \frac{2}{v-2} = 2\sqrt{3}$, solving this equality (which turned out to be extremely unpleasant) yields the result that $v = \sqrt{3} + 1$.

   Ans: $\sqrt{3} + 1$

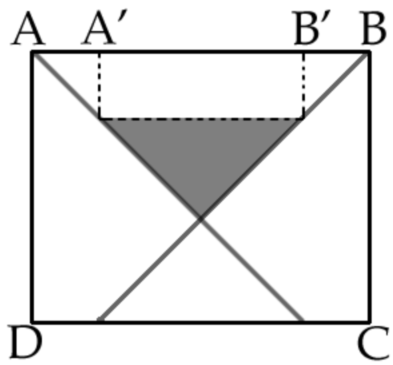
7. Consider the case where you have 100 \$1 dollar coins. Obviously, this is not a permissible combination of 100 coins, since there are no \$2 or 50¢ coins. However, we can obtain a legitimate combination if we change one of those \$1 dollar coins to a \$2 dollar coin, as well as two \$1 dollar coins to 50¢ coins. If we repeat this series of changes, we obtain yet another combination of 100 coins that total to \$100. This may be done at most 33 times, since there aren't enough \$1 coins to change. It should be clear that we have covered all possible arrangements, and so:

   Ans: 33

8. Since we know that the equation is true for all x, it must be true for x=1. In which case, letting x=1 yields: $a+b+c+d=(2-2)\times(1+4)-(1-2)\times(1-5+4)=0$

   Ans: 0

9. The region that we want is the shaded region shown in the diagram below. We know that AD is folded to AB', therefore the length of $B'B$ must be $\frac{5}{4}-1=\frac{1}{4}$. Similarly, AA' must also be $\frac{1}{4}$ long. This means that $A'B'$ is $\frac{3}{4}$ long and utilising the many 45° angles in the picture, we can easily determine that the area of the shaded triangle is $\frac{1}{2}\times\frac{3}{4}\times\frac{3}{8}=\frac{9}{64}$.



   Ans: $\frac{9}{64}$

10. Notice that because this is an arithmetic sequence, $a_1+a_{47}=a_3+a_{45}=a_5+a_{43}=\ldots=a_{23}+a_{25}$. Therefore, the average of all the odd terms is just the average of $a_{23}+a_{25}$, that is: a_{24}. Similarly, the average of all 47 terms of the sequence is also a_{24}. We know that there are odd-numbered terms in the sequence, so $a_{24}=\frac{1272}{24}=53$. Therefore, the sum of all the terms must be $53\times 47=2491$.

    Ans: 2491

11. It should be common knowledge that any number whose digits add up to a multiple of 9, is itself a multiple of 9. (e.g.: 7+7+4=18, so 774, 747 and 477 are all divisible by 9.) Moreover, any number whose last two digits are divisible by 4, must be divisible by 4. (e.g. the last two digits of 122393928 is 28, which is divisible by 4, therefore 122393928 is divisible by 4.)

    Given this knowledge, and noticing that $36=4\times 9$, we can easily determine if the original number was divisible by 36 by testing to see if it was divisible by 4 and 9.

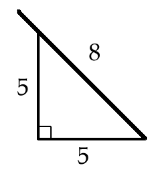
    Since the last two digits of the number is 64, therefore 347_47_64 must be a multiple of 4.

    Therefore, we only need to determine when it's divisible by 9. This is of course when the two missing digits add up to either 1 or 10. There are eleven different ways to do this: (0,1) (1,0) (1,9) (2,8) (3,7) (4,6) (5,5) (6,4) (7,3) (8,2) and (9,1).

    And as there are $10\times 10=100$ possible combinations for those two spaces in total, the probability that the original number was divisible by 36 is $\frac{11}{100}$.

    Ans: $\frac{11}{100}$

12. You should know from year 10 maths (failing that, year 11 advanced general maths), that the area of a triangle is equal to $\frac{1}{2}AB\sin(\theta)$, where A and B are two sides of a triangle and θ is the angle in between those two sides. Taking A=5 and B=5 shows that we can maximise the area of our triangle if $\sin(\theta)$=1, that is, if $\theta=90°$. Therefore, the biggest possible area that these three sticks can enclose is a right-angled triangle with two sides equal to 5 metres and the hypotenus equal to $5\sqrt{2}$ metres.



Therefore, the area is $\frac{1}{2} \times 5 \times 5 = \frac{25}{2}\text{m}^2$.

Ans: $\frac{25}{2}\text{m}^2$

13. There are $\binom{6}{2} = 15$ ways of choosing two of the six vertices. Since you can draw two equilaterals triangles with each pair of chosen vertices, this means that we have a total of 30 equilateral triangles. However, we have two sets of 3 equilateral triangles that are exactly the same (have fun figuring out which ones), therefore we must subtract 4 from the total.

    Ans: 26

14. Every positive integer (apart from 1) can be expressed as a product of its prime factors. (e.g.: $999 = 3^3 \times 37^1$) Let's say that our number N can be written as $p_1^{k_1} \times p_2^{k_2} \times p_3^{k_3} \times \ldots \times p_m^{k_m}$, the the number of positive factors of N will be $(k_1+1)(k_2+1)(k_3+1)\ldots(k_m+1)$. This is because, any factor of N, when expressed as a product of its own prime factors, can have $0, 1, 2 \ldots k_1$ powers of p_1 and $0, 1, 2 \ldots k_2$ powers of $p_2 \ldots$ etcetra. Knowing this, we can see that we'd like to have $(k_1+1)(k_2+1)(k_3+1) = 20 = 5 \times 2 \times 2 \Rightarrow k_1 = 4, k_2 = 1$ and $k_3 = 1$. Picking the smallest possible corresponding primes, we have $2^4 \times 3^1 \times 5^1 = 240$. Note that we have not rigorously shown that 240 is the correct answer, because some assumptions were made. The actual proof is thus left as an exercise.

    Ans: 240

15. Let the number of goals and behind be g and b respectively. Therefore, we want to figure out all possible integer solutions to the following equation:

    $gb = 6g + b \Rightarrow b(g-1) = 6g$

    Notice that $g = 0$ and $b = 0$, or $(g, b) = (0, 0)$ is clearly a solution to this equation.

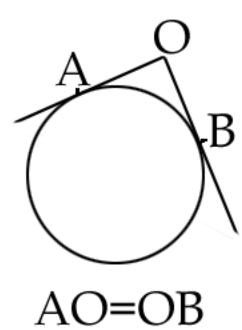
    Having covered the $g = 0$ case, we can say that; since g and $g-1$ are relatively prime, $g-1$ is must be a factor of 6.

    $\Rightarrow g - 1 = 1, 2, 3, 6 \Rightarrow g = 2, 3, 4, 7 \Rightarrow (g, b)$=(2,12) (3,9) (4,8) (7,7)

    As you can see, there are five different combinations possible.

    Ans: 5

16. Hint: Notice that all four sides of the quadrilateral enclosing the circle are in fact tangential to the actual circle. Therefore, you'll want to use the following property of tangents, illustrated in the diagram below.



Ans: 18

17. Hint: 2 lines can intersect at most once, 3 lines can intersect at most 3 times and 4 lines: 6 times. See if you can figure out a pattern and then try to use it to solve the problem.

    Ans: $m = 64$, $k = 5$

18. Hint: Google the word iteration.

    Ans: 1003

19. Hint: The three digit base-8 number $abc_8 = 64a+8b+c$ and when reversed, $cba_8 = 64c+8b+a$. And remember to convert it into base-10 after you're done.

    Ans: 189

20. Hint: There are a many ways of doing this. One of the nicest methods, involves breaking up this infinite sum into infinite number of infinite sums that add up nicely.

    Ans: 4

21. Hint: Remember that you need not work out the actual number of classes at each school, only the difference between those two figures.

    Ans: 2

22. Hint: Try the problem using smaller triangles and see if you can notice any patterns.

    Ans: 2005! (That's a factorial sign, not an exclamation mark.)

23. Hint: Again, there are different methods to do this question. The nicest way involves you finding out more about 'expected values' and looking up university level 'Probability' textbooks.

    Ans: $3\frac{3}{4}$

24. Hint: Look up 'transition matrices' in the book you borrowed for the question above.

    Ans: $\frac{1}{7}$

25. Hint: Please don't try this one at home.

    Ans: 78400