1. Say you bought x shells. The whole price is halved, so you pay x/2. This is only 3 more than what you would have paid, which is (x-12). So x/2 = x-12+3, so x/2 = 9 and x = 18.

2. Substitute r = h = π into the formula $\pi * r^2 * h$ to get π^4

3. The middle (fourth) number is 91/7 = 13, so the lowest is 13 - (4-1) = 10.

4. There are 7 months of this year with 31 days, 4 with 30 days and 1 with 28 days (2007 is not a leap year). The months with an even number of days will have the same number of even days of the month as they have odd days of the month. The 7 months with 31 days however, will have one extra odd date (16 odd dates and 15 even dates). This year therefore has 7 more odd dates than even dates.

5. Maurice eats all 5 (Joanna always lies)

6. Let the item cost x cents, before tax. 0.04*x = x/25 is a whole number, so 25 divides (is a factor of) x. When the tax is added to the item, its cost is x/25 + x (cents) = 26x/25 cents. This is a whole number of dollars, so 100 divides 26x/25. Then 50 divides 13x/25. As 50 and 13 have no common factors (greater than 1), we must have that 50 divides x/25. Then 50*25 = 1250 divides x. The smallest such x is 1250, so the cost of the item (before tax) is 12.50.

7. 3^22, 32^2, 2^23 and 22^3 are all extremely large, so clearly such expressions won't be involved in attaining the lowest possible value. $3^2 * 223$ and $322 * 2^3$ are therefore the only reasonable candidates. The former is equal to 2007, and the latter 2576. We choose the lower one, 2007.

8. Say you tell them the integer x, and they want to find a real number y so that xy = x-2y, i.e. y(x+2) = x. If x is not -2, then they can simply pick y= x/(x+2). They can't do this if x = -2 however, as the denominator would be 0. If you pick x= -2, then they have to satisfy the equation -2y = -2 - 2y, i.e. 2 = 0, which is impossible. Hence, -2.

9. If you do not have four of the same suit, then the maximum number of cards you can have is 4*3 (since there are 4 suits and you have at most 3 of each) = 12. However, you have 13 cards (contradiction), which means that you always have 4 of the same suit. Thus the probability is 1.

10. Rectangle ACBD, so that AC = BD = 16, and AD = BC = 12. Suppose and fold along AC (we are left with the isosceles trapezium ABCD). Now by Pythagoras' Theorem, CD = $v(12^2 + 16^2)$ = 20. Let AC and BD meet at E, and let BE = AE = x, so that EC = 16 - x. Now, by Pythagoras, $x^2 + 12^2 = (16 - x)^2$, so $x^2 + 144 = 16^2 - 32x + x^2 = x^2 - 32x + 256$. Now $32x = 256 - 144 = 112$. Thus x = 112/32 = 7/2 = 3.5. The perimeter in question is equal to $12 + 12 + 20 + 2x = 44 + 7 = 51$.

11. 6 mathematicians, 3 pushpots and 2 woodchucks, so 11 in total.

12. For simplicity say there are 100 cars. 20 of them are red, and let's say there are x red convertibles. Then x/20 = .3, so x = .3*20 = 6. If there are C convertibles then x/C = 6/C = .25 = 0.25. Now C = 6/0.25 = 24. So there are 20 red cars and 24 convertibles, but we have counted 6 of them twice (the ones that are both red and convertibles). So there are 20+ 24 - 6 = 38 cars that are red and/or convertibles. Now there are 100 - 38 = 62 cars that are neither red nor convertiblesthat's .62 of our total number of cars.

13. If the first die is a 1 then the other two need to sum to 10 (3 possibilities). If the first die is a 2, then the other 2 need to sum to 9 (4 possibilities). If the first die is a 3, the other two need to sum to 8 (5 possibilities). If the first die is a 4, then the other two need to sum to 7 (6 possibilities). First die 5, other two 6 (5 possibilities); first die 6, other two 5 (4 possibilities). Total number of ways is 3+4+5+6+5+4 = 27.

14. The pattern is: /3, +3, x3, -3 and repeat, so our answer is 13.5/3 = 4.5

15. Remove all of the cubes that are sides of edges, but not corners, and remove the very centre cube. This leaves 14 cubes, none sharing a face with any other. Our surface area is 14*6 = 84.

16. $1/2 + 1/3 + 1/7 + 1/42 = 1$...so the ultimate answer is 42.

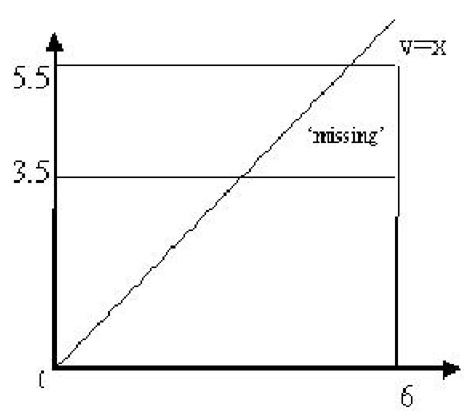
17. Let the circles centred at A, B have radius 5, circle centred at C have radius 8. Let the circles centred at A and B touch at F, the circles centred at B and C meet at D, the circles centred at A and C meet at E. Let the fourth circle be centred at G and have radius r. Note firstly that A, E, C are collinear (lie in a line); B, D, C collinear; F, G, C collinear. BC = 5 + 8 = 13 and BF = 5, so by Pythagoras FC = 12. Now FG = 12 - CG = 12 - (8 + r) = 4 -r. Also, BG = 5 + r. Now apply Pythagoras to triangle BFG: $(4 - r)^2 + 5^2 = (5 + r)^2$ so r = 8/9.

18. Define $f(x, y) = xy - x - y + 2 = (x - 1)(y - 1) + 1$. Notice that $f(1, y) = 1$. So Kim can never get 1 off the whiteboard. The number remaining at the end must therefore be 1.

19. Let the number in the top-left corner be x. Then the overall sum is 115 + x. Find expressions for the bottom-left corner, the middle square, and the others in terms of x by equating the row, column and diagonal sums with 115. Finally, x = 200 so that the sum in question is 315.

20. For margin to be 3, one of them has to win the first 3 gamesprobability is $2 * 0.5^3$ (since either of them can do it) = 1/4. For margin to be 1, they have to be tied 2-2 after 4 games (doesn't matter who wins last one), and the probability of this is $4C2 * 0.5^4 = 3/8$. Then $Pr(margin2) = 1 - Pr(margin1) - Pr(margin3) = 1 - 1/8 - 3/8 = 3/8$. Now average winning margin is $3/8 * 1 + 3/8 * 2 + 1/4 * 3 = 3/8 + 3/4 + 3/4 = 15/8$.

21. Let 8:59.30 be time 0. Let the time you arrive on the 2nd platform after taking 30 seconds to cross be 'x'. Let the time the 2nd train leaves be 'y'. You know that 0=x=6 and 3.5=y=5.5. Let the rectangle defined by these inequalities be R. Now, 'missing' the train corresponds to the inequality y=x. The probability of missing the train will be the proportion of the area of R which satisfies this inequality. Plot y against x. R has area $6 * 2 = 12$. The region corresponding to 'missing' has area (2x2) + x 2 = 3. So, the probability of missing the train is 1/4.



22. We ultimately want to work out the area 'covered' by the towers. Take square ABCD. Each tower has a maximum range which is part of a circle. Let the maximum ranges of the towers at D and C meet at X, and let Y be the altitude from X to CD (i.e. the point on

CD so that XY is perpendicular to CD). DY = 1/2 and $DX = 1/\sqrt{3}$, so by Pythagoras, $XY = 1/(2\sqrt{3}) = 1/2 * DX$. Now there are several ways to see that $\angle XDY = 30$ (one way is to let X' be the reflection of X across CD, and now XDX' is equilateral). Let the minimum ranges of the towers at A and D meet at Z. Now $\angle ADZ = \angle XDC = 30$, so now $\angle XDZ = 30$. We can now work out the area of sector XDZ to be $0.5 * (1/\sqrt{3})^2 * (\pi/6)$ the formula is $0.5 * radius * angle$ (in radians) $= \pi/36$. There are four such areas, so their total area is $\pi/9$. Add to this the area of the four triangles CXD, AZD etc, which is $4 * 0.5 * 1 * 1/(2\sqrt{3}) = 1/\sqrt{3}$, and our total area (i.e. the probability that a unit is detected, since ABCD has area 1) is $\pi/9 + 1/\sqrt{3}$.

23. Let the dog's post be B, the first sheep's post be A, the second sheep's post be C. Let D be the point so that ABCD is a rectangle. Let E, F be such that A is the midpoint of ED and C is the midpoint of DF. Draw a circle of radius 4 centred at A (call it C_1) and a circle of radius 3 centred at C (call it C_2). Draw a circle of radius 5 centred at B (call it C_0). Note that C_1 passes through D,E; C_2 passes through D,F; C_0 passes through D,E,F. We want the sum of the areas in C_1 or C_2 but not C_0. We can see that there is no area within C_1 AND C_2 but not C_0. So we can just add the area of C_1 C_0 (call this area A_1) to the area of C_2 C_0 (call this area A_2) . If we let $\angle DBE = a$ (note that now $\angle DBF = \pi$ - a radians), we can work these out by cleverly adding and subtracting areas. Now $A_1 = 0.5 * \pi * 4^2 + 0.5 * 8 * 3 - 0.5 * 5^2 * a$, and $A_2 = 0.5 * \pi * 3^2 + 0.5 * 6 * 4 - 0.5 * 5^2 * (\pi - a)$. So $A_1 + A_2 = 24$(all the π's cancel).

24. There are $\binom{100}{2} = 4950$ possibilities for the two poisoned cakes. We can assign each of these a number in binary (from 0000000000001 to 1001101010110). The first person goes to all the cakes with a 1 in the first digit, the second person goes to all the cakes with a 1 in the second digit etc. Some of the people will die, and they will give away the binary representation of the 2-cake combination. We require 13-digit binary representations, and hence 13 people.

25. We can deduce that $1 \to 4, 3 \to 0, 2 \to 8, 8 \to 2, 7 \to 3$. We can also deduce that 9 and 4 are programmed to 5 and 6 in some order (we don't know which). That leaves 0, 5, 6 to be programmed to 1 ,7, 9 in some order. Now '46 + 30 + 95' = 10('4+9') + 10('3') + '0 + 5 + 6' = 10(5+6) + 10*0 + (1+7+9) = 127.

Solutions provided by Sam Khai-Ho Chow.