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The Editor’s Column

Welcome to Issue Three of this glorious mathematical publication we call Paradox. Since Kristijian’s tyrannical reign over this editorial column has ended, I would like to open the doors on a new, more democratic Paradox.

Before I introduce the articles presented herein, I would like to invite members of the MUMS community who have until this point remained silent, to rise up, get out your computer, phone, tablet or carrier pigeon and contribute to the production of Paradox by emailing me at paradox.editor@gmail.com. We are always on the lookout for feature article writers, joke makers, editors and proof-readers, typesetters, puzzle creators and whatever other position you may feel will enhance the pleasure that is reading Paradox.

I am also interested in starting up a Letters section, so please forward any comments or correspondence that you have about the articles in this issue of Paradox, or about MUMS events you’ve attended or other mathematical pursuits that you may believe the wider readership will be interested in. Whether this is a success or not will emerge from the size of the Paradox mailbox over the coming semester.

In addition to this, I would like to start up a regular Research column in Paradox to keep our readership informed of new progress and exciting innovations happening in Mathematics and Statistics both at the University of Melbourne and in other institutions. Two ways I thought this could happen are either having a regular Columnist or Editor presenting a synthesis of results or having individual research students submitting short (probably around 1000-word) summaries of their research. If anyone is interested in either taking up an editorial role in this area or sharing their research with the wider MUMS community, please email me at paradox.editor@gmail.com.

Enough on that now, here are some previews of what you will see when you stop reading this editorial drivel:

This issue contains biographies on your new MUMS Executive Committee – who they are, what they look like and what they seek to do with their newfound (or re-invigorated, in the case of returning committee members) power. You can learn more about what one of the lecturers from then Department of Mathematics and Statistics does when he’s not teaching in Ruwan Devasundara’s exclusive interview with Professor Barry Hughes.
This issue has (unintentionally on my part) become the Erdős issue, with two articles, one a biography on his life and the other a review of a documentary that follows the life and work of the most prolific mathematician in history.

Furthermore, Dougal Davis continues to tie our brains up in knots with the third installment of his popular Knot series, former MUMS President Damjan Vukcevic tells us about odds and the Paradox Puzzle section is revived with a mathematically-infused Cryptic Crossword.

I hope you enjoy this issue of Paradox and I wish you all the best for this semester.

— Ben Hague

Words from the President

Dear readers of Paradox,

If you’ll turn to the inside of the front cover, you’ll notice many new names for this year’s committee. Even I had not met some of the committee members before the AGM. Nonetheless, MUMS continues on with its usual schedule of events.

This semester, in addition to our weekly seminars, keep an eye out for the University Maths Olympics, an hour-long problem solving competition which also involves some ”running” (or, at least, runners).

We held a games night on the second Friday of semester after our regular weekly seminar, and many people dropped by and played board and card games, such as Settlers of Catan, Dominion, and Small World. By the way, you can come and play these games whenever enough people with enough spare time are gathered in the MUMS room.

While on the topic of the MUMS room, if you drop by around 12:30 p.m. each day, you’ll find a group of people doing the daily trivia quiz from The Age and/or The Australian.1

— Mel Chen

1And in case you wonder what all the numbers next to names are on the whiteboard, they’re to keep track of people’s points.
Interview with Barry Hughes: Mathematician and Educator

Professor Barry Hughes is a well-known lecturer at the University of Melbourne. He graciously set aside some time to speak to me about his mathematical journey.

You did your BSc at the University of Queensland and your PhD at ANU. Could you tell me more about your education and how you came to be in the position you are in today?

I grew up in Queensland, and for most of my education I went to ordinary suburban schools. Though I have no complaints about the teaching that I received, I wouldn’t say the education was particularly enriching or inspiring with regard to mathematics. It was simply competent. Things got more interesting for me when the family moved back to Brisbane in 1971 for my Year 11, and I got to attend a selective government high school, Brisbane State High. There, the intellectual material I was presented with was ramped up considerably, and after a short period of adjustment, I found that very rewarding and it probably led me to where I am today.

It’s interesting to reflect that the only piece of assessment I ever failed at school or university was my first Year 11 in-class maths test, and the shock of failing that changed the way I thought about mathematics and the way I studied it. In particular, it shifted me from learning how to do things to trying to understand the meaning of what I was doing, and educationally that was very formative.

Like many people of my generation, I was not particularly careerist in motivation - indeed, the idea of going to university for people of my generation was almost novel, and yet I had this vague ambition to attend university after school. I enrolled in the science degree, having no particular aspiration for a professional career directly, and in my first year studied chemistry, physics and double maths. Though I respect chemists, the experience of first-year chemistry laboratory was sufficiently distasteful that I dropped chemistry and devoted myself to mathematics and physics in second year, and then to mathematics alone (emphasising the applied side) in third year and for honours.

Having spent a summer as a vacation scholar at ANU between third year and the honours year, I returned there for my PhD in applied mathematics, and wrote my thesis on the applications of continuum mechanics in colloid
science, drawing on some of my undergraduate exposure to elasticity theory and fluid mechanics, but also building on strengths in complex analysis and related techniques.

As a PhD student I also had my first exposure to a biologically-related research problem, perhaps foreshadowing my more recent research interests. After completing my thesis, I was fortunate to get a postdoctoral fellowship sponsored by the CSIRO to spend some time in America (working with physicists at the University of Rochester and the University of Maryland), and following that, I took a short summer job with Rockwell in California and then completed a two year postdoc in chemical engineering at the University of Minnesota.

So up until that stage, I saw myself more as a scientist whose work had a distinct mathematical flavour than a mathematician specifically. But subsequently after moving back to Australia and working as a post-doc at ANU, a mathematics lecturer at the Royal Military College Duntroon, and finally as a mathematics lecturer at the University of Melbourne, I describe myself as a mathematician. In a way, I have become more overtly mathematical since then, but my interests are very much in applied mathematics.

**You are the author of a two-volume book, Random Walks and Random Environments. How would you explain these two mathematical concepts to the uninitiated?**

I tell my graduate students that you should be able to explain at the bar or at the pub, in a few sentences to anyone, roughly what it is you do and why it’s interesting, without getting too technical. So, I will describe to you briefly two models, and talk about their applications.

First, a random walk: you can think of an individual who is optimally mathematically drunk, such that they are so drunk as to be completely confused about where they are going, but not so drunk that they can’t move. Then you ask the question, if such a person departs from a point, what can I say about where they will be in the future, and what they will have done in the interim? You can use ideas like this to describe diffusion processes, motion of microorganisms, cell motion in developmental biology, and a whole lot more.

The second problem concerns the random environment. Imagine a network of streets, say a nice rectangular grid, and I’m a random council worker and I travel down every street and I roll dice to decide whether or not I’m going to
dig up the street. I go through the city once, visiting each street only once, and once I’ve done that, I’ve disrupted a number of streets that stops traffic from flowing along them. Then I ask the question, is it possible for one person to drive from one side of the city to the other? That’s clearly a statistical question, because if I dig up a relatively small fraction of the streets then it is most unlikely that I would have created a barrier that can’t be crossed. If on the other hand I dig up ninety percent of the streets, then it’s quite unlikely that the motorists will be able to drive all the way through. This idea leads to percolation theory, which is one of the simplest models for a random environment.

So, the theory of random environments is to do with a variety of models where you impose some disorder on space, and you consider what the geometry of that disorder is, or you consider processes that take place that are affected by this disorder. If you were interested in the properties of porous materials, the mechanical structure of composites, or the design of gas masks, those are the kinds of models that you would want to pursue. And of course if you want real fun, you perform a random walk in a random environment. It’s like a postmodern play in which the script is random (each actor rolls dice to decide what to say or do next) and the stage setting is randomly rearranged before every performance.

**Having worked at many different institutions, what spurred you to become interested in teaching?**

Well, if you want to stay in the business of academia, or practice as a non-industrial mathematician, then you need either an academic position, or a position in a research institute. My first permanent job, at the Royal Military College Duntroon, came with substantial teaching duties, and I traded that up for a position at Melbourne, which also came with teaching duties. I believe that if you take on a teaching position you ought to take it seriously. And so, I have devoted quite a lot of time to my teaching, perhaps more than I would have if I were solely concerned with my research goals or research productivity.

**Yes, I’ve seen during my own studies that you write your own detailed material for your subjects.**

I find it hard to teach from other people’s material; not because I don’t like what they’ve done, but because it doesn’t come with my voice. And I think it’s important that when you deliver a subject you have your own perspective on it – or to be a little bit deeper. You have to understand what the key underlying
ideas are. So, for example, I’ve taught Accelerated Maths 2 now since 2008, and each year I teach it some fundamental themes become clearer to me.

In some sense there is the notion of a big picture; people tend to think of maths courses as being a collection of results or a series of tricks to be learned, but maths is really about ideas, and the view that I’ve come to with advanced age is that the important thing to teach is the basic ideas. In teaching those, there are results and tricks that come up, but you have to have the right conceptual framework, and then it all makes sense; otherwise, you just remember a bunch of stuff that may or may not come up in the exam, and you end up forgetting anyway. By understanding the fundamental ideas, it is much easier to remember the content, or recreate it as needed, and you have been through some intellectual development in doing so as well.

**Given your interest in the interface between school and university mathematics, do you think students enter university adequately equipped for the mathematic proficiency expected in tertiary education?**

I am very critical of the current regimes of assessment of mathematics in high schools. I’m very comfortable with the rather draconian scheme that I went through in Queensland many years ago, where your end of Year 12 exams covered a two-year syllabus, and both of the maths subjects had two three-hour exams, which were the only assessment. Though this may seem intimidating and high-pressured, you actually have to have a proper understanding of a whole two-year syllabus, and given the length of the exams, it is possible for the examiners to assess you thoroughly on many areas of the syllabus, and also to ask some questions that require a depth of understanding.

This contrasts with the short duration of the examinations that school students sit here, where they may even be permitted to bring in graphics calculators and formula sheets as well. I feel that these don’t deliver good value. They tend to cultivate an attitude towards assessment in which students feel like there is only a bunch of tricks they need to learn, without a need to know what they are actually doing. It tends to encourage students to merely swot things at the last minute, or survive solely by training on past papers, whereas students should be taking the time to reflect and understand what is going on, preparing them to grow into the mindset of tertiary education. The fundamental learning aspect is by far the most important, and it is under-rated by almost all students.
Students that attend my subjects will know that I have my own personal spin on education, and I also try and emphasise the anecdotal history of the subject as important. Some of the asides I make in my lectures about history and culture are there for a very deep purpose: I think it’s tragic that students can complete a maths major and have no idea about the historical period of almost any mathematical figure they’ve ever heard of.

Apart from being a lecturer at Melbourne University, you are also a prolific researcher, having covered many different areas of research. Could you tell us a bit about what you’re currently working on?

I have very eclectic interests, and over a long career I’ve changed the direction of my interests on a number of occasions. Though I continue to do other things as well, for about 10 years now I’ve been working with my department colleague Kerry Landman and our post-docs and graduate students in mathematical biology and related areas. Some of the work we do is distinctly biological. On other occasions, we investigate interesting mathematical questions that we were led to consider from an original biological context; biology has often given us a reason to study these interesting problems.

A lot of our work is related to biological movement, such as cell motility in developmental biology, but we’re also interested in what I would call big questions, an example of which is: how does the random movement of individuals, whose movements affect each other, manifest in large scales. If you had some kind of crowd, and you took your glasses off and viewed them from afar, you would notice some kind of flow. Then we ask, what is the relation between the local rules that the crowd obeys (which may have both deterministic and random elements) and what you see from a distant perspective. There are many contexts in which these kinds of stochastic problems arise.

Out of the vast topics that you’ve been able to work on, is there any that stands out?

I always tend to return to things that are random-walk-ish. I spent about 11 years of my life writing those two books on that topic, but there are other pieces of work that I am proud of, sometimes because they contain an element of mathematical sneakiness that brings me some quiet satisfaction.

You have often mentioned the beauty of complex analysis. What is it about that topic that intrigues you the most?
I can give you several answers. As an undergraduate at UQ, third year complex analysis was a subject that I really got; it was particularly well taught, and by the end of that subject I was confident that I had understood it completely. I fell in love with the subject, and subsequently in my PhD, there were a number of occasions where I had to use a proper understanding of this subject. Also, during my first research success as a postdoc, I relied on my use of complex analysis, and on a number of occasions since then, complex analysis has saved me. So, for that pragmatic reason, I have enormous affection for it. In addition, it’s just a beautiful subject: there is a kind of completeness to complex analysis that no other mathematical subject has for me.

Finally, is there any advice you would give to students in their final year of study, or to students that might be interested in entering research or academia?

I’ll answer a slightly different question to the one that you have asked: my general advice to students of mathematics is to try and engage with the content of the subject, and try and understand what it means. Then when you graduate, whether become a professional mathematician, or a professional user of mathematics (in academia or elsewhere) or do something entirely unrelated to mathematics, if you’ve actually engaged with the subject and you have developed your mind appropriately, then you will find that the skills that you have learned will serve you well in your future career, whatever it is.

Consider the British Empire, which was run by classics scholars from Oxford and maths graduates from Cambridge. Now it is not clear that studies in classics or mathematics specifically fit anybody for colonial administration, but nonetheless, it doesn’t seem to have done too badly. In American terms, going to college is where you develop your mind, and maths is a good way to develop your mind whether you use maths subsequently or not.

— Ruwan Devasurendra

An infinite number of mathematicians walk into a bar. The first mathematician asks for 1/2 a beer, the second asks for a 1/4 of a beer, the third asks for an 1/8th of a beer. Just as the next mathematician is about to place his order the barman says "know your limits boys" and pours one beer.
Getting to Know the New MUMS Committee

At the AGM last semester some people were elected by the MUMS electorate to represent them on the committee and to make decisions about MUMS, organise MUMS events and keep MUMS running smoothly. I asked the Executive Committee to write something about ourselves so you get to know us better:

**Mel Chen, President**
Hello, I’m Mel, current Trea-, I mean, President, of MUMS.¹ I’m currently in my third semester of a Master’s in Pure Maths, and my research topic is the Isomorphism Problem for Coxeter groups.²

I became interested in MUMS after reading an issue of Paradox, and later joined up since I hung around the room so much, playing board games such as Settlers of Catan.

I ran for President since I was one of two members from the old committee who were staying on. I hope to guide the new committee in the running of MUMS, so that they have experience to pass down after I, too, leave.

Apart from maths, I pass my time playing video games, reading, and occasionally playing piano.

**Jenny Fan, Vice-President**
I’m currently in my second-year of Actuarial Studies and the only other committee member from last year apart from Mel who’s not graduating... yet!

Over the past year, I’ve thoroughly enjoyed serving MUMS as the Education Officer. It has been great getting to know so many new faces around the Maths Department but most of all, I enjoy giving back to the MUMS community and helping them run the events that I’ve enjoyed.

In my spare time, I enjoy playing chess, singing musical theatre, playing piano and running regularly. In addition to this, I’ve been reading Paradox since high school, and it has always been full of interesting articles, funny quotes

¹Sorry, habit!
²If that means anything to you.
and exciting puzzles!

As VP, I aim to help the President run the MUMS events smoothly, and also pitch in from time-to-time to keep the website updated.

Sarah Sims, Secretary
I’m currently in my second year studying a Bachelor of Science and intending to major in statistics. I joined MUMS last year and started going to the seminars and reading Paradox, and these both really unleashed my inner maths nerd!

I decided to run for secretary as I have a knack for things being organised and in order and thought it would be a great opportunity to meet more people who also enjoy a good math joke.

When I’m not hiding out under a tree or crunching numbers I can be found eating vast amounts of ice-cream and frozen yoghurt, playing board games and trying to escape from the maths building and enjoy as much sunshine as I can!

Declan El-Hage, Treasurer
I’m a first year commerce student who likes maths and people who like maths and is eager to spend more time playing around with and enjoying maths.

If in your studies you find something interesting and want to share it with someone I’m always keen to listen!

Ben Hague, Paradox Editor
I’m in my third year of my Bachelor of Science majoring in Atmosphere and Ocean Science and studying the concurrent Diploma of Mathematical Sciences, specialising in Statistics and Stochastic Processes. I play the tuba in a community concert band and enjoy doing cryptic crosswords and playing Scrabble.

I became involved in MUMS because I was interested in attending the seminars, and attend these when I can, and also because the members were friendly to First Years. I also enjoy using my penchant for useless facts to write questions for the trivia night, after I completed in the first incarnation of the “Table of Arts Students” (of whom none studied Arts) team in First Year.

As the Editor of Paradox I aim to provide you, the reader, with four issues in this coming financial year, with this copy you are currently perusing being
the first. I will work with Damian to publish some seminar summaries (these will start in the next issue) and keep persuading people to write beautiful, stimulating and accessible articles — without sounding like I am advertising a brothel!

**Damian Pavlyshyn, Education Officer**

I’m now in my third year of a maths degree with a focus darting between pure maths and probability. When not deciding which of these topics interest me more, I love hiking and going to the theatre.

The MUMS seminars caught my eye in first year, and I went to a few, then started turning up at the MUMS room. Finally, having taken part in one of the excellent MUMS trivia nights, I realised that there was no longer any hope of escape.

In my capacity as Education Officer, I organise the seminars and buy the all-important food and drink. I hope to expand the seminar repertoire to include mathematicians from institutions such as Monash and WEHI, and to start following the most interesting seminars up with short *Paradox* articles.

**Paul Nguyen, Publicity Officer**

I like math! I am very excited to be part of the new committee and about what I can offer MUMS. I got involved with MUMS due to my love of math! And because of the friendly environment that the MUMS club offered.

I ran for committee so that I could contribute and be apart of something that was much bigger than I was. MUMS was about maths, and maths is something I care about. In addition, being part of MUMS has also assisted me in doing better at university and has provided me with heaps of help as to what I could achieve with my degree in Mathematics.

I am currently in my second year studying a Bachelor of Science, majoring in Mathematics. My interests are guitar, indie music, soccer and applied mathematics. I hope you guys see my MUMS posters around town!

— The 2014–2015 MUMS Executive

**Q.** What does the ’B’ in Benoit B. Mandelbrot stand for?

**A.** Benoit B Mandelbrot
Paul Erdős (1913–1996)

Another roof, another proof

Having published more papers than any other author in history, Erdős was a Hungarian-born mathematician famous for his extensive contribution to mathematical literature. He was also known for his unique lifestyle and vocabulary, and his knack for arriving on the doorstep of a colleague and announcing, “My brain is open”, before spending time collaborating with them “until he was bored or his host was run down, and then [moving] on to another home”.

Early Life

Erdős was the third and sole surviving child of Lajos Erdős and Anna Wilhelm, whom had tragically lost two daughters to scarlet fever by the time they brought Paul home from the hospital. In face of such loss, the two doting parents focused much of their attention upon Erdős, and being mathematics teachers themselves, were thrilled to discover that “by the age of four [he] was a full-blown mathematical prodigy”: at that age, he could multiply three- and four-digit numbers in his head; took pleasure in mentally calculating the number of seconds a person had lived; and later told of his joyous “independent discovery” of negative numbers when a house guest asked, “What is 100 minus 250?”.

Philosophy

Erdős had a distinctive perspective on life. He often referred to the SF, or the “Supreme Fascist”, the Number-One Guy Up There, God, who was always tormenting Erdős by hiding his glasses, stealing his Hungarian passport, or, worse yet, keeping to Himself the elegant solutions to all sorts of intriguing mathematical problems. Erdős believed that all these beautiful solutions were kept by the SF in a tome which he referred to as “the Book”. Thus, Erdős commenting on one’s proof as being “straight from the Book” was the highest compliment one could receive.

2Ibid
4Ibid., 27-29.
5Hoffman, 4.
His language was also a reflection of his mathematical, and at times eccentric, philosophy on life:

– “When did the misfortune of birth overtake you?” (When were you born?)

– “epsilons” (little children)

– “bosses” (women) and “slaves” (men)

– “captured” (married), “liberated” (divorced) and “recaptured” (re-married)

– “noise” (music)

– “poison” (alcohol)

– “preaching” (lecturing on mathematics).6

Erdős also was known to be somewhat of a mathematical vagrant, as he spent much of his life hopping from one colleague’s house or university to the next, collaborating and preaching to his heart’s content. Materialistically, all he truly cared about were his mathematical notebooks; he carried one of these notebooks at a time with him on his travels, and left the remainder in the care of his close friend and collaborator Ronald Graham.7 The notebook, his whole wardrobe and his beloved radio would all fit inside a small suitcase, which was usually all that accompanied him. Sometimes, he would have a plastic department store bag with him, brimming with mathematical papers of interest, but the bulk of his own papers and letter correspondence was managed on his behalf by Graham himself.

He never married, and rarely indulged in activities other than mathematics, despite the many efforts of his colleagues to introduce him to other hobbies and pastimes. As one author put it: “Erdős was a mathematical monk. He renounced physical pleasure and material possessions for an ascetic, contemplative life, a life devoted to a single narrow mission: uncovering mathematical truth”.8

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6Ibid.,8

7Graham’s number is named after him, after he used the extraordinarily large number in a proof.

8Hoffman, 25.
Legacy and Death

It has been said that Erdős has co-authored with at least 485 people\textsuperscript{9}, and his colleagues took this opportunity to casually quantify the degree of academic ‘closeness’ between a given individual and Erdős himself by what they termed an individual’s ‘Erdős number’. Erdős had an Erdős number of 0, whilst those in the group of 485 co-authors are said to have an ‘Erdős number’ of 1. Anyone who has co-authored with an individual of ‘Erdős number’ of 1 would themselves have an ‘Erdős number’ of 2, and so on. Anyone who was sufficiently well-removed from the mathematical sphere was given and ‘Erdős number’ of infinity. In good humour, some mathematicians have investigated the properties of Erdős’ ‘Collaboration Graph’, the graph whose vertices are the 485 collaborators, and whose edges indicate a publication between any two authors. There even exists a website dedicated to the analysis of this entity.\textsuperscript{10}

Erdős’ cardiac health began to deteriorate in the year of 1996, and he passed away from a heart attack by September. Ahead of time, he had prepared an epitaph for himself, written in Hungarian, reading “Finally I am becoming stupider no more”.\textsuperscript{11}

Despite never winning a Fields Medal, Erdős did win the Wolf Prize in 1984, and true to his frugal lifestyle and generous spirit, he donated much of the prize money to a scholarship he established in his parents’ names.\textsuperscript{12} During his lifetime, Erdős had offered money to those who could solve particular mathematical problems set by him. These were later known as Erdős’ Problems\textsuperscript{13}, and currently the awards are being managed by Graham and his wife, Fan Chung.

Erdős’ legacy is legendary in that it is twofold: not only has he contributed to “number theory, combinatorics, probability, set theory and mathematical analysis”\textsuperscript{14} via his aforementioned conjectures, but the impact that he has had on the social practice of mathematics has been felt for generations to come.

— Ruwan Devasurendra

\textsuperscript{9}Ibid.,13
\textsuperscript{10}See http://www.oakland.edu/enp/.
\textsuperscript{11}Hoffman, 3.
\textsuperscript{12}Ibid.,10
\textsuperscript{13}For a full list, see http://en.wikipedia.org/wiki/List_of_conjectures_by_Paul_Erdős.
\textsuperscript{14}Taken from the ceremonial description on his Wolf Prize.
Odds

Introduction

Sometimes I hear people ask, “what are the odds?” and without skipping a breath they start mentioning probabilities rather than odds.

Unless you have been taught otherwise, I guess it’s natural to think of these as vaguely the same sort of thing (not to mention similar concepts such as chance and likelihood). However, within statistics they have precise and different meanings.

You can think of probability and odds as being like Celsius and Fahrenheit. They measure the same thing but on different scales. You need to convert from one to the other before you can compare them.

A probability is a number between 0 and 1 that we use to represent how likely something is to happen. I think most people know this and use it correctly. The letter p is often used to denote a probability.

Odds are another way to give a number to an event to represent its chance of occurring, but this time the scale goes between 0 and infinity. A probability of p corresponds to an odds of \( p/(1-p) \). For example, suppose there are only two possibilities for the weather tomorrow: rainy or sunny. If the probability of rain is 0.2, then the corresponding odds of rain is \( 0.2/0.8 = 0.25 \). The probability of being sunny is therefore 0.8, and the odds is \( 0.8/0.2 = 4 \).

This version of odds is also called odds in favour. An alternative is odds against, which is simply the reciprocal, \( (1-p)/p \). In the weather example, the odds against rain is 4, and the odds against sun is 0.25.

Gambling odds

The place where most people encounter odds is at the horse races (or other sports where gambling is popular). In this case, the ‘odds’ are a way for bookmakers to show how much money they would pay if you were to select the winning bet. The favourite horse in a race will pay less than any of the others, because it is deemed most likely to win.

It turns out many different types of odds are used. (I don’t know why there are so many, it only makes things difficult!) They typically vary by country. I’ll describe three of them.
In the UK, the standard is fractional odds. It is the ratio of the winnings to the bet amount, expressed as a fraction. For example, if were offered odds of 3/2 for the horse Prancing Diva, and placed a $10 bet, you would be paid 3/2 x $10 = $15 if it won, plus also your original $10 bet, leading to a total payout of $25. If the horse loses, you lose your $10 bet.

In Australia, the convention is to use decimal odds. This is the ratio of the full payout (winnings plus original bet) to the bet amount, expressed as a decimal. For Prancing Diva, the equivalent in decimal odds is 2.5 (which is $25 divided by $10). It is easy to calculate decimal odds from fractional odds, simply convert to a decimal and add 1 (for the original bet).

In the USA, the system of choice is moneyline odds. It is represented as a whole number. When positive, it is the winnings for a bet of 100. When negative, it is the bet required to win 100. The odds for Prancing Diva in this case would be 150. To get moneyline odds from fractional odds, simply multiply by 100 if greater than 1, and multiply the reciprocal by -100 if less than 1. For example, 3/1 becomes 300 and 1/3 becomes -300.

The table compares these three types of odds (see next page).

**Gambling Odds vs True Odds**

Strictly speaking, gambling odds are different to the odds I described earlier, which I’ll refer to as true odds. Whereas true odds are precise statements about how likely an event is, gambling odds describe possible financial transactions on offer. You can think of them as showing the ’cost’ of various bets.

To understand the relationship between the two, suppose there will be a race between Prancing Diva, Gallop-a-lot, Canterberry and Trotskyite, with the the probability of each horse winning being 0.5, 0.2, 0.2 and 0.1 respectively. The corresponding true odds against are 1, 4, 4 and 9.

Honest Joe the bookmaker could offer fractional odds of 1/1, 4/1, 4/1 and 9/1 on this race (i.e. equal to the true odds against). If he were to do this, his average profit would be exactly zero. This is not a good way to run a business. Instead, he offers the less favourable odds of 2/3, 3/1, 3/1 and 7/1. This reduces his required payouts and increases his profit.

If we pretend these are the true odds against and convert them back to probabilities, we get 0.6, 0.25, 0.25 and 0.125 respectively. We call these the implied probabilities.
Probabilities have the nice property that if you add them all up, you always get 1. In Joe’s case, the total is 1.225. The difference is due to him building in a profit margin. The bigger the difference, the greater the profit.

An intuitive explanation is that Joe is ‘pretending’ each horse is more likely to win than they actually are, so that he can pay you less on your bets. This requires him to add in extra probability, pushing the total over the true total probability of 1.

While gambling odds need to reflect the underlying knowledge of how likely each of the possible events are (otherwise punters would have a sure bet), the

<table>
<thead>
<tr>
<th>Gambling odds</th>
<th>Comparison values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fractional</td>
<td>Moneyline</td>
</tr>
<tr>
<td>10/1</td>
<td>1000</td>
</tr>
<tr>
<td>9/1</td>
<td>900</td>
</tr>
<tr>
<td>5/1</td>
<td>500</td>
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<td>4/1</td>
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<td>3/1</td>
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<tr>
<td>2/1</td>
<td>200</td>
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<tr>
<td>3/2</td>
<td>150</td>
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<td>1/1</td>
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<td>2/3</td>
<td>-150</td>
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<tr>
<td>1/9</td>
<td>-900</td>
</tr>
<tr>
<td>1/10</td>
<td>-1000</td>
</tr>
</tbody>
</table>
The fact that they don’t respect the probability sum property means they are not true odds.

**Everyday odds**

When your friend asks you, “what are the odds?” and starts quoting numbers, which type of odds are they (if any at all)? Most likely it’s unclear. To avoid confusion, I like to stick with probabilities. What do you prefer?

— Damjan Vukcevic

(reproduced with author’s permission from: http://damjan.vukcevic.net/2014/01/21/odds/)

**Film Review: N is a Number: A Portrait of Paul Erdős (1993)**

*Dir: George Paul Csicsery*

*First released in 1993; first broadcast on television in 1995.*

Paul Erdős (born 1913, died 1996) is a name no doubt familiar to most Paradox readers. The Hungarian mathematician was a prolific paper-writer (1525 papers)\(^1\) and collaborator (511 collaborations)\(^2\) and gave rise to the concept of ‘Erdős numbers’ – think ‘six degrees of Kevin Bacon’ in the mathematical world!\(^3\)

*N is a Number* is an hour-long documentary that examines Erdős’ life and work. It consists of a loose narrative structured around interviews with Erdős and his friends and colleagues from all over the world. The focus of the documentary is clearly on Erdős the man rather than on the substance of his contributions to maths.

The maths in the film is sparse and fairly basic, but the documentary is crammed with interesting tidbits about Erdős’ lifestyle, working habits and

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\(^1\)http://www.oakland.edu/enp/pubinfo/.

\(^2\)http://www.oakland.edu/enp/thedata/.

\(^3\)The American Mathematical Society provides a Erdős number calculator on its website: see http://www.ams.org/mathscinet/collaborationDistance.html.
views of the world. The interviews with friends, colleagues but especially Erdős himself paint a portrait of a brilliant, eccentric and kindly mathematical nomad.

There are many light moments – for example, Erdős playing table tennis, or doing tricks for ‘epsilon’ (his term for children), or describing life as a game with ‘the Supreme Fascist’ (God). But there are also hints of some of Erdős’ personal struggles: the death of friends in Hungary during World War II, the loss of his mother, and his occasional loneliness even when surrounded by people.

As can be expected for a documentary filmed over 20 years ago, the footage is quite grainy and the narration can sometimes feel a little stilted. But the extensive footage of Erdős talking about his work and his life, broken up into digestible chunks and organised thematically throughout the film, makes it all worthwhile.

Given Erdős’ stature in the field, this documentary is a must-see for all budding mathematicians. It has also piqued my interest in Erdős’ life, and I will be adding Paul Hoffman’s biography of Erdős, *The Man Who Loved Only Numbers*, to my to-read list!

— Julia Wang

Knot 3 (with apologies to Lewis Carroll)

“How about we play a little guessing game?” said the witch sweetly. “I’ll go and fetch some hats: some of them will be white and the rest of them will be black. Then you’ll all line up one in front of the other, and I’ll pop a hat on each of your heads. Starting from the back, I’ll ask each of you what colour your hat is: when I ask you, you can answer ‘black’ or ‘white’, but otherwise no talking or it’s not fair. Those of you who are right can leave, and the rest of you will help me with my dinner!”

With that the witch left the room and suddenly the gnomes could move again. Some of them ran to the door, and some to the window where they had come in, but all exits were sealed tight.

“What will we do?” wailed the fat gnome. “I don’t fancy being made into a pie!”

Whilst the other gnomes were panicking, the most mathematically inclined
gnome sat down against a leg of the table and thought, what could the
gnomes do to save as many as possible? Of course, the gnome at the back
would have to take his chances, since none of them would be able to see his
hat, but what of the others? Could the gnome at the back communicate the
colours of each of the other gnomes’ hats by saying only ‘black’ or ‘white’?
The gnome was about to discount this as impossible, when another thought
occurred to him: each gnome would know the colours of the hats in front of
him and, if they found a strategy that worked, the colours of the hats
between himself and the gnome at the back. With that, the clever gnome
realised what they must do!

The gnome jumped to his feet and shouted for silence until the frightened
gnomes calmed down enough to listen to him, as he told them of his plan.

“This is what we must do: whoever ends up at the back of the line must count
the number of white hats he sees in front of him. If he sees an even number,
he must say ‘white’ and if he sees and odd number, he must say ‘black’. For
the rest of us, when it is our turn to guess, we count how many white hats
there are in front of us, and how many have been guessed behind us,
excluding the gnome at the back. Since we know whether the total number is
even or odd, this will tell us whether our own hat is black or white!”

After thinking about it for a short while, the gnomes agreed that this was a
very good plan, although none of them wanted to be at the back of the line.

When the witch returned, she gazed intensely at the gnomes, and they felt
their legs move of their own accord, walking them into place in a straight
line. The witch then walked along the line, placing a pointed witch’s hat on
the head of each gnome she passed. She returned to the gnome at the back,
which as luck would have it was the mathematically inclined gnome, and
stared menacingly into his eyes.

“Now”, she said, “Guess!”

The gnome did as he had told the others and counted the number of white
hats ahead of him. He counted four, so he guessed “black”. The witch’s eyes
registered the barest flicker of annoyance as she moved down the line to the
next gnome and repeated the command. As she continued through each of
the gnome, her irritation became more and more apparent as each gnome
guessed correctly, until she was seething with anger as the final gnome
uttered the colour of his hat.
As luck would have it, the clever gnome’s hat was indeed black, so every gnome had guessed correctly! The witch had no choice but to release them, as witches cannot break their word. However, she felt that the gnomes still deserved some punishment for eating her pie, even if she could not eat them herself. So as the gnomes ran for the now unlocked door, the witch muttered a spell under her breath, followed by the slightest flicker in the light outside. In their haste, the gnomes did not notice their surroundings until they had all run a fair distance from the door.

The first gnome to notice yelled in surprise, and stopped dead, only to be bowled over by the gnome behind him. By the time he had gotten to his feet, the other gnomes had noticed it too, and were looking around in amazement. Rather than being in the forest as they had expected, they found themselves on a stony ridge overlooking deep gorges on either side. Behind them they could see no trace of the door through which they had come, only an outcropping of twisted black rock which would surely prevent any gnome from passing it along the ridge. Ahead the ridge sloped gradually upwards towards a mountainous peak, from which emanated a faint red glow which illuminated the sky above. Foul-smelling vapours hissed from fissures in the rocks above and below them, so that all but the brightest stars in the night sky above them were obscured by a cloud of pollution. The moon was just beginning to rise, and was stained a hideous orange by the unclean air.

As they stared in dismay at their forbidding surroundings, the most far-sighted of the gnomes—who had found the fishing spot that very morning that seemed so long ago, as well as the witch’s cottage that had led them to this disaster—spotted a cave mouth in the side of the mountain ahead of them. The gnomes all agreed to shelter in the cave for the night and look for a way down in the morning. None of them mentioned the thought that was foremost in each of their minds: even if they could escape this accursed place without falling to their deaths among the sharp rocks below, how could they ever find their way home?

The climb to the cave was not difficult, but the gnomes were glad to rest their feet when they reached it. The baleful moon which had watched them all the way was unsettling, and the gnomes did not like being exposed on the ridge, feeling as if a bird of prey could swoop down and carry them off at any moment.

Before they settled down for the night, the gnomes felt that they should first explore the full extent of the cave. Who knew what dangerous animals lived
in such a desolate place? Besides, they wanted to be sure that there were no vents in the cave that would suffocate them with polluted air during the night.

To their surprise, the cave tunneled far back into the mountainside, the air becoming warmer and warmer the deeper they went. The gnomes were just about to turn back, reasoning that no animal would hide this far back in the cave when it was so hot, when the sharp-eyed gnome noticed a faint reddish glow emanating from further down the tunnel. He immediately wanted to investigate, but the other gnomes held him back, feeling that they had gotten into too much trouble already that day to go running after mysterious lights in caves. However, there was something strangely intriguing about the tunnel, and after much arguing, curiosity got the better of the gnomes and they advanced cautiously towards the glow.

Now as it happened, there was a dangerous animal living somewhere at the back of that cave; in fact, an animal much more dangerous than anything the gnomes expected. For this mountain was no mere volcano, but the fiery home of a dragon! This particular dragon, who lay ensconced on a mound of gold in the cavern to which the gnomes were creeping, had been aware of the gnomes from the moment they had emerged from the witch’s house onto the ridge outside his home. It was through the power of his will that the gnomes had failed to see the rough hewn steps leading down from the ridge which the people in the village on the far side of his valley used to bring him his food, and it was at his subliminal urging that the gnomes had decided to investigate the glow at the back of the tunnel. For dragons hunt from the comfort of their lairs by ensnaring their unwitting prey with their minds as much as by swooping down on them with the terror of their flaming breaths and razor-sharp teeth and claws.

The dragon—whose name I am bound to tell you is Gaurinth, lest he hunt me down for not giving him the personal credit he is due for his role in this tale—had recently taken to keeping prisoners in small cave adjoining his main cavern. He would amuse himself either by watching them as they tried to escape, edging past the lake of molten rock which barred the entrance to his prison only to run desperately back as Gaurinth slithered out from wherever he was hiding, or by setting them riddles or puzzles, eating those who failed and leaving the rest for later.

When gnomes emerged into the dragon’s lair, they at first did not notice Gaurinth as he lay on the heap of gold at the centre of the cavern, his power
keeping his fearsome bulk from their attention. As they wandered in amazement around the cavern, the dragon quietly guided them with his thoughts until he lay between them and the exit. Then he roared with delight and raised himself to his full height.

The gnomes gazed in terror at this fearsome beast. They were still more surprised to when the dragon stopped roaring and said in a smooth but commanding voice, “Come out of your hole, my prisoners, and meet your newest addition!”

At this, steam began to pour from a cave mouth behind the gnomes, and five dirty and hungry people emerged, coughing and spluttering, onto a ledge of rock before a pool of molten rock. The gnomes stared at them, but quickly returned their attention to the dragon, who was looking at them with what they could swear was an expression of amusement. For Gaurinth had noticed the black and white hats the gnomes still wore, and, as he was personally acquainted with the witch, who often came in for a chat whilst collecting mushrooms that grew around his volcanic home, he made a decent guess of the puzzle the witch had set them.

“Well”, thought the dragon to himself, “that’s a nice idea, but I can do one better!”

“Well now”, said the dragon to his prisoners, “our new friends have given me a wonderful idea for a puzzle! I’m going to ask one of my slaves in the village to bring me sixteen hats tomorrow morning, each a different colour. Once we have the hats, you will all line up in single file, and I will place a hat on each of your heads, and keep the remaining one to myself. Starting from the back, you must each guess the colour of your hat out loud and say nothing more. If you guess wrong, you shall become part of my breakfast. If you are right, I promise I won’t eat you until at least the following day. Now, off with you!”

With that, the dragon swept up the gnomes in one massive paw and tossed them into the cave, into which the other prisoners were already retreating.

“Sixteen colours!” groaned one of the gnomes. “How will we get out of this one? We barely managed with two!”

To be continued...
Paradox Puzzles

Welcome to the new Puzzles page, which this week features a cryptic crossword compiled by the editor. Readers are welcome to submit their own puzzles to this page via paradox.editor@gmail.com.

Solution is supplied, see last page.
Upcoming MUMS Events

15 AUGUST     Seminar: Dr Owen Jones
22 AUGUST     Seminar: Dr Guogi Qian
29 AUGUST     Seminar: Dr Andrew Robinson
5 SEPTEMBER  Seminar: Joe Chan
12 SEPTEMBER Seminar: Presenter TBA
19 SEPTEMBER Seminar: Dr Alex Ghitza
27 SEPTEMBER Seminar: Presenter TBA
10 OCTOBER    Seminar: Dr Richard Brak
17 OCTOBER    Seminar: Presenter TBA
24 OCTOBER    Seminar: Presenter TBA

MUMS Trivia Night

Solution to Cryptic Crossword:

Why did the mathematician go to the brothel?
He was trying to find a solution to f(x)=0.

Paradox would like to thank Ruwan Devasurendra, Dougal Davis, The 2014-5 MUMS Executive, Mel Chen, Damjan Vukcevic and Julia Wang for their contributions.