Look up the definitions of the following objects in the book or the internet:
map (also called “function”), source (also called “domain”), target, image, graph, (composition of two maps) well-defined, injective (also called “one-to-one”), surjective (also called “onto”), bijective (also called “one-to-one correspondence”).

(1) (4 points): Write down the definitions of injective and surjective.

(2) (More negations):
Complete the following sentences
(a) (9 points): A relation $f \subset X \times Y$ is not a (well-defined) map if and only if . . .
(b) (4 points): A map $f$ is not surjective if and only if . . .
(c) (8 points): A map $f$ is not injective if and only if . . .
(d) (3 points): A map $f$ is not bijective if and only if . . .

(3) (10 points): Consider the map
$$f: \mathbb{N} \rightarrow \mathbb{N}$$
$$x \mapsto 2x.$$  
(Here we used the notation “$x \mapsto 2x$” for “$f(x) = 2x$”.)
(a) What is the image of $f$?
(b) Is $f$ surjective?
(c) Is $f$ injective?

(4) (20 points): Let $X$ and $Y$ be the sets
$$X = \{a, b, c, d\}$$
$$Y = \{a, b, e\}$$

Draw the graph of each of the following, decide whether it is a well-defined map and if so, whether it is injective, surjective or
bijective: (It is possible that several or none of these hold.)

\[ f: X \rightarrow Y \]
\[ a \mapsto b \]
\[ b \mapsto e \]
\[ c \mapsto a \]
\[ d \mapsto b \]

\[ g: X \rightarrow Y \]
\[ b \mapsto e \]
\[ c \mapsto a \]
\[ d \mapsto b \]

\[ h: Y \rightarrow X \]
\[ a \mapsto b \]
\[ b \mapsto a \]
\[ e \mapsto a \]

\[ i: Y \rightarrow X \]
\[ a \mapsto b \]
\[ b \mapsto c \]
\[ a \mapsto a \]
\[ e \mapsto d \]

\[ j: Y \rightarrow X \]
\[ a \mapsto b \]
\[ b \mapsto c \]
\[ e \mapsto d \]

(5) \textbf{(12 points)}: Try to formulate for a general relation what the property “is a (well-defined) map” means for its graph and for a map \( f \) what the properties, “injective”, “surjective” or “bijective” mean for the graph.

(6) \textbf{(30 points)}: Let \( f: X \rightarrow Y \) and \( g: Y \rightarrow Z \) be injective maps. Prove that this implies that \( g \circ f \) is also injective. (You will do the analogous proof for surjective maps in class on Monday.)