Euclidean vectors and coordinates

Ways to picture the vectors $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ -2 \end{bmatrix}$
Adding vectors

\[
\begin{bmatrix}
1 \\
3
\end{bmatrix} = \begin{bmatrix}
-2 \\
1
\end{bmatrix} + \begin{bmatrix}
3 \\
2
\end{bmatrix}
\]

Arrange your vectors tip to tail!
Linear transformations

send lines to lines and preserve the origin
Algebraically

Since vector addition is described using parallelograms, a transformation $T$ of Euclidean space is linear if and only if

\[
T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})
\]

\[
T(a \cdot \vec{v}) = a \cdot T(\vec{v}).
\]

Linear transformations are exactly those transformations that preserve sums and multiplication by scalars.

This formalism generalises to abstract vector spaces.
The matrix of a linear transformation

In the above picture:

\[
\begin{bmatrix}
1 \\
0
\end{bmatrix} = \begin{bmatrix} \ \ \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 \\
1
\end{bmatrix} = \begin{bmatrix} \ \ \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
2 \\
3
\end{bmatrix} = \begin{bmatrix} \ \ \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
4 \\
-1
\end{bmatrix} = \begin{bmatrix} \ \ \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y
\end{bmatrix} = x \begin{bmatrix} \ \ \\
\end{bmatrix} + y \begin{bmatrix} \ \ \\
\end{bmatrix} = \begin{bmatrix} \ \ \\
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}
\]