Different ways to describe a plane in $\mathbb{R}^3$

Cartesian Equation

$$2x - 4y + 6z = 8$$

Parametric equation

$$x = 4 + 2s - 3t$$
$$y = s$$
$$z = t$$

Vector Equation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$$

Three Point Form

through $\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$, and with unit normal vector

$$\frac{1}{\sqrt{56}} \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$$

through $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
How to switch between these descriptions

**Cartesian equation to Parametric equation**: like you have practiced when solving linear systems

**Cartesian equation to unit normal vector form**: read off unit normal vector from the coefficients and find one solution of the equation

**Unit normal vector form to Cartesian equation**: the normal vector gives the coefficients, its dot product with the stationary vector gives the constant term.

**Vector equation to unit normal vector**: take the cross product of the direction vectors and normalize it (i.e., divide by its length).

**Parametric equation to vector equation and back**: very easy

**Three point form to vector equation**: subtract one of the vectors from the other two to get the direction vectors

**Anywhere to three point form**: Find three solutions that do not lie on a line
Different ways to describe a line

Vector Equation

\[
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} = \begin{pmatrix}
  2 \\
  1 \\
  3
\end{pmatrix} + t \begin{pmatrix}
  7 \\
  -3 \\
  1
\end{pmatrix}
\]

Two point form

Through \( \begin{pmatrix}
  2 \\
  1 \\
  3
\end{pmatrix} \) and \( \begin{pmatrix}
  9 \\
  -2 \\
  4
\end{pmatrix} \)

Parametric Equation

\[
x = 2 + 7t \\
y = 1 - 3t \\
z = 3 + t
\]

Cartesian Equation

\[
\frac{x - 2}{7} = - \frac{y - 1}{3} = z - 3
\]

Switching is easy. To switch from a parametric equation for a line to a Cartesian equation for the same line, solve for \( t \).
Intersections

Whenever we ask you to find the intersection of two things, we are asking you to solve a linear system.

If you are trying to intersect two planes and the system has no solution, they are parallel (but not identical). Similarly, a line could be parallel to a plane or intersect it, two lines can either intersect or be parallel or skew, etc.

Careful: the definition of angle between two planes or two lines in the official slides is ambiguous.

For a linear system in three variables, our intuition for how many solutions we expect coincides with our intuition for intersecting planes in $\mathbb{R}^3$. 