1. Suppose that students answer questions on a test and that a specific student has an aptitude $T$. A particular question might have difficulty $d_i$ and the student will get the answer correct only if $T > d_i$. Now if we consider $d_i$ fixed and $T \sim N(\mu, \sigma^2)$, then the probability that a randomly selected student will get the answer wrong is $p_i = P(T < d_i)$.

Show how you might model this situation using a probit regression model.

**Solution:** We have

$$p_i = P(T < d_i) = P \left( \frac{T - \mu}{\sigma} < \frac{d_i - \mu}{\sigma} \right) = \Phi \left( \frac{1}{\sigma} \frac{d_i - \mu}{\sigma} \right)$$

which is in the form of a probit regression model with predictor variable $d$, $\beta_0 = -\mu/\sigma$ and $\beta_1 = 1/\sigma$.

2. The dataset **discoveries** lists the number of great scientific discoveries for the years 1860 to 1959, as chosen by “The World Almanac and Book of Facts”, 1975 Edition. Has the discovery rate remained constant over time?

To answer this question, fit a poisson regression model with a log link, and use the deviance to compare a null model with models including the year and year squared as predictors.

**Solution** First we fit two models, the first including the year and the second the year and the year squared. The plot gives the fitted rates in each case.

```r
> data(discoveries)
> disc.df <- data.frame(year=1860:1959, disc=discoveries)
> model1 <- glm(disc ~ year, family=poisson, disc.df)
> summary(model1)

Call:  
glm(formula = disc ~ year, family = poisson, data = disc.df)

Deviance Residuals:
   Min     1Q Median     3Q    Max
-2.8112 -0.9482 -0.3533  0.6637  3.5504

Coefficients:
            Estimate Std. Error   z value Pr(>|z|)
(Intercept) 11.354807   3.775677  3.00701  0.00264 **
year       -0.005360   0.001982 -2.70509  0.00683 **

---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for poisson family taken to be 1)

  Null deviance: 164.68 on 99 degrees of freedom
Residual deviance: 157.32 on 98 degrees of freedom
AIC: 430.32

Number of Fisher Scoring iterations: 5

> model2 <- glm(disc ~ year + I(year^2), family=poisson, disc.df)
> summary(model2)
```

```r
```
Call:
glm(formula = disc ~ year + I(year^2), family = poisson, data = disc.df)

Deviance Residuals:
   Min      1Q  Median      3Q     Max
-2.9066 -0.8397 -0.2544  0.4776  3.3303

Coefficients:
                Estimate Std. Error z value Pr(>|z|)
(Intercept)  -1.482e+03  3.163e+02  -4.685  2.79e-06 ***
year          1.561e+00  3.318e-01   4.705  2.54e-06 ***
I(year^2)   -4.106e-04  8.699e-05  -4.720  2.35e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 164.68 on 99 degrees of freedom
Residual deviance: 132.84 on 97 degrees of freedom
AIC: 407.85

Number of Fisher Scoring iterations: 5

> x <- disc.df$year
> plot(x, disc.df$disc)
> beta1 <- model1$coefficients
> lines(x, exp(beta1[1] + beta1[2]*x), col="blue", lty=2)
> beta2 <- model2$coefficients
> lines(x, exp(beta2[1] + beta2[2]*x + beta2[3]*x^2), col="red")
From the plot both year and year squared look significant, but we need to quantify this observation. For a poisson model the deviance only looks $\chi^2$ if the responses are large enough to look vaguely normal, which they are not in this case. None-the-less, we can use deviance differences to perform likelihood ratio tests. From the above, the null model has deviance 164.68, the model with just year has deviance 157.32, and the model with year and year squared has deviance 132.84. We test the significance of adding year and then year squared:

```r
text(chisq.test(164.68-157.32, 1, lower.tail=FALSE))
```

```r
text(chisq.test(157.32-132.84, 1, lower.tail=FALSE))
```

There is strong evidence that year improves the model, and very strong evidence that year squared has something to add. We conclude that there is strong evidence that the discovery rate has changed over time.

3. The ships dataset from the MASS package gives the number of damage incidents and aggregate months of service for different types of ships broken down by year of construction and period of operation. Load the dataset using the commands `library(MASS)` then `data(ships)`.

Develop a model for the rate of incidents (i.e. a poisson regression model with log link), describing the effect of the important predictors.

**Solution** After loading and inspecting the data, it seems that year and period are really ordered factors rather than numerical predictors, so we alter these variables appropriately.
Next we explore the relations between the variables. All the variables look important, and we note that applying a square root transform to `service` improves the relation between `service` and `incidents`.

We can fit now a log-poisson model. From the Wald tests each variable looks significant. We could confirm this using likelihood ratio tests based on the deviance.
Deviance Residuals:

    Min   1Q Median   3Q  Max
-2.1509 -1.2833 -0.7905  0.2751 2.6875

Coefficients:

    Estimate Std. Error z value Pr(>|z|)
(Intercept) 0.207853 0.234122  0.888  0.374649
   typeB   -0.121206 0.250163 -0.485  0.628024
   typeC   -1.005644 0.329657 -3.051  0.002284 **
   typeD   -0.574643 0.289933 -1.982  0.047481 *
   typeE   -0.025521 0.236667 -0.108  0.914127
  year.L    0.654626 0.194109  3.372  0.000745 ***
  year.Q   -0.822592 0.122829 -6.697 2.13e-11 ***
  year.C   -0.128340 0.097295 -1.319  0.187142
period75   0.726592 0.125831  5.774 7.73e-09 ***
  rootserv  0.021648 0.002202  9.830  < 2e-16 ***

---

Signif. codes:  0 '***'  0.001 '**'  0.01 '*'  0.05 '.'  0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 730.253 on 39 degrees of freedom
    Residual deviance: 67.035 on 30 degrees of freedom
    AIC: 184.9

Number of Fisher Scoring iterations: 5

Note that because year is ordered R has used linear, quadratic and cubic contrasts. You can see them exactly using contrasts

> contrasts(ships$year)

    .L .Q .C
[1,] -0.6708204 0.5 -0.2236068
[2,] -0.2236068 -0.5 0.6708204
[3,]  0.2236068 -0.5 -0.6708204
[4,]  0.6708204  0.5  0.2236068

Next we look for interactions.

> model1 <- glm(incidents ~ type + year + period + rootserv + type:year, family=poisson, ships)
> pchisq(deviance(model) - deviance(model1), df.residual(model) - df.residual(model1), lower.tail=FALSE)

[1] 0.0001099668

> model2 <- glm(incidents ~ type + year + period + rootserv + type:period, family=poisson, ships)
> pchisq(deviance(model) - deviance(model2), df.residual(model) - df.residual(model2), lower.tail=FALSE)

[1] 0.08820292

> model3 <- glm(incidents ~ type + year + period + rootserv + type:rootserv, family=poisson, ships)
> pchisq(deviance(model) - deviance(model3), df.residual(model) - df.residual(model3), lower.tail=FALSE)

[1] 0.003187932

> model4 <- glm(incidents ~ type + year + period + rootserv + year:period, family=poisson, ships)
> pchisq(deviance(model) - deviance(model4), df.residual(model) - df.residual(model4), lower.tail=FALSE)

[1] 0.0001018208

> model5 <- glm(incidents ~ type + year + period + rootserv + year:rootserv, family=poisson, ships)
> pchisq(deviance(model) - deviance(model5), df.residual(model) - df.residual(model5), lower.tail=FALSE)

[1] 0.0001018208
> model6 <- glm(incidents ~ type + year + period + rootserv + period:rootserv, family=poisson, ships)
> pchisq(deviance(model) - deviance(model6), df.residual(model) - df.residual(model6), lower.tail=FALSE)

[1] 0.4123239

> model7 <- glm(incidents ~ type + year + period + rootserv + type:year + period:year, family=poisson, ships)
> pchisq(deviance(model1) - deviance(model7), df.residual(model1) - df.residual(model7), lower.tail=FALSE)

[1] 0.0005265296

> model8 <- glm(incidents ~ type + year + period + rootserv + type:year + period:year + type:rootserv, family=poisson, ships)
> pchisq(deviance(model7) - deviance(model8), df.residual(model7) - df.residual(model8), lower.tail=FALSE)

[1] 0.07904069

> model9 <- glm(incidents ~ type + year + period + rootserv + type:year + period:year + year:rootserv, family=poisson, ships)
> pchisq(deviance(model7) - deviance(model9), df.residual(model7) - df.residual(model9), lower.tail=FALSE)

[1] 0.8730395

> summary(model7)

Call:
  glm(formula = incidents ~ type + year + period + rootserv + type:year + period:year, family = poisson, data = ships)

Deviance Residuals:
  Min       1Q   Median       3Q      Max
-1.80944  -0.00785  -0.00005   0.00847  2.06533

Coefficients:  
Estimate Std. Error z value Pr(>|z|)     
(Intercept)  -9.556e+00  3.042e+03  -0.003 0.997     
typeB         5.649e+00  2.668e+03   0.002 0.998     
typeC         3.815e+00  2.668e+03   0.001 0.999     
typeD         -5.655e+00  4.666e+03  -0.001 0.999     
typeE         -3.376e-01  3.779e+03  -0.000 1.000     
year.L        1.206e+00  8.164e+03   0.000 1.000     
year.Q        -2.107e+01  6.085e+03  -0.003 0.997     
year.C        -1.162e-01  2.721e+03  -0.000 1.000     
period75      5.714e+00  1.463e+03   0.004 0.997     
rootserv     -1.426e-02  1.341e-02   1.063 0.288     
typeB:year.L  -1.467e+01  7.158e+03  -0.002 0.998     
typeC:year.L  -1.451e+01  7.158e+03  -0.002 0.998     
typeD:year.L  4.069e+00  1.043e+04   0.000 1.000     
typeE:year.L  -1.669e+00  1.014e+04   0.000 1.000     
typeB:year.Q  1.019e+01  5.335e+03   0.002 0.998     
typeC:year.Q  1.040e+01  5.335e+03   0.002 0.998     
typeD:year.Q  1.028e+01  9.333e+03   0.001 0.999     
typeE:year.Q  -1.288e+00  7.559e+03   0.000 1.000     
typeB:year.C  -4.192e+00  2.386e+03  -0.002 0.999     
typeC:year.C  -5.540e+00  2.386e+03  -0.002 0.999     
typeD:year.C  -1.430e+01  8.091e+03  -0.002 0.999     
typeE:year.C  2.112e-01  3.380e+03   0.000 1.000     
year.L:period75 1.364e+01  3.926e+03   0.003 0.997     
year.Q:period75 1.044e+01  2.926e+03   0.004 0.997     
year.C:period75 4.229e+00  1.309e+03   0.003 0.997

(Dispersion parameter for poisson family taken to be 1)

  Null deviance: 730.25 on 39 degrees of freedom
  Residual deviance: 10.53 on 15 degrees of freedom
AIC: 158.4

Number of Fisher Scoring iterations: 18

Curiously, although the type:year and period:year interactions are significant, none of the Wald tests are significant in the model with interactions. This suggests dependency between our predictors. We look for a more parsimoneous model using `step`.

```r
> model10 <- step(model7)
Start: AIC=158.4
incidents ~ type + year + period + rootserv + type:year + period:year
Df Deviance AIC
- rootserv 1 11.694 157.56
<none> 10.530 158.40
- type:year 12 45.965 169.83
- year:period 3 28.151 170.02
Step: AIC=157.56
incidents ~ type + year + period + type:year + year:period
Df Deviance AIC
<none> 11.694 157.56
- year:period 3 72.163 212.03
- type:year 12 123.483 245.35
> summary(model10)
Call:
glm(formula = incidents ~ type + year + period + type:year +
    year:period, family = poisson, data = ships)
Deviance Residuals:
       Min        1Q    Median        3Q       Max
-1.86294  -0.03467  -0.00005   0.03221   2.18897

Coefficients: Estimate Std. Error z value Pr(>|z|)
  (Intercept) -9.0814    3030.6260  -0.003    0.998
  typeB       6.9806     2733.6364   0.003    0.998
  typeC       3.7125     2733.6364   0.001    0.999
  typeD      -5.7949     4742.5185  -0.001    0.999
  typeE      -0.5124     3865.9456  -0.000    1.000
  year.L      0.7141     8132.0230  -0.000    1.000
  year.Q    -21.2793    6061.2520  -0.004    0.997
  year.C    -0.1840     2710.6743  -0.000    1.000
  period75   5.5950     1308.4060   0.004    0.997
  typeB:year.L-16.0763    7335.1161  -0.002    0.998
  typeC:year.L-15.0317    7335.1161  -0.002    0.998
  typeD:year.L   4.0042    10660.4227   0.001    0.999
  typeE:year.L  10.5783     9485.0369   0.001    0.999
  typeE:year.Q -1.3731     7731.8913  -0.002    0.999
  typeE:year.C   0.1601     3457.8069  -0.000    1.000
  year.L:period75 14.9779    3510.8217   0.004    0.997
  year.Q:period75 10.2033     2616.8120   0.004    0.997
  year.C:period75 4.1899     1170.2739   0.004    0.997
```
We see that, given the type:year and period:year interactions, rootserv is no longer significant. Formally, the reason for this is that rootserv itself can be predicted using type, year, period, type:year and period:year, so it is no longer needed when it comes to predicting incidents. Having said that, there is a clear scientific reason for wanting rootserv in the model, so given that the AIC for model10 is not much smaller than that for model7, I would be inclined to keep it.

The fact that the individual parameters in model10 are all close to zero is not necessarily a problem, but does suggest that some of these levels could be grouped. Testing that two levels of a factor are the same is not as easy for a glm as for a linear model, but can still be done indirectly using likelihood ratio tests. What we have to do is fit a model where the levels are combined, and then see if it performs significantly worse.

4. The infert dataset from the survival package presents data from a study of infertility after spontaneous and induced abortion. Using a logistic regression model, analyse and report on the factors related to infertility based on this data. (Don’t use the factor stratum, as it is confounded with the other predictors.)

**Solution**

The response is case, with 1 indicating infertility and 0 fertility. The data comes from a case-control study, the aim of which was to estimate the effect of the number of prior induced and spontaneous abortions on the probability of becoming infertile. In the original study it was believed that education, age and parity (something numeric, whatever it is) were confounding variables, so the cases were separated into 83 strata based on these variables, and two controls were recruited from each stratum. (One control from one of the strata was subsequently omitted from the dataset, for reasons unexplained.)

Because of how the data were collected, the observations are not independent, so a logistic regression model is not actually appropriate. None-the-less we will carry on as if it is, and next week will analyse the data using a conditional logistic regression.

```r
> library(survival)
> data(infert)
> modell <- glm(case ~ age+parity+education+spontaneous+induced,
+               data = infert, family = binomial())
> summary(modell)

Call:
  glm(formula = case ~ age + parity + education + spontaneous +
      induced, family = binomial(), data = infert)

Deviance Residuals:
          Min         1Q     Median         3Q        Max
-1.76030  -0.81621  -0.49559   0.83487   2.65358

Coefficients:  Estimate Std. Error z value Pr(>|z|)
(Intercept)   -1.14924    1.41220  -0.814  0.4158
age            0.03958    0.03120   1.269  0.2046
parity        -0.82828    0.19649  -4.215  2.49e-05 ***
education6-11yrs -1.04424    0.79255  -1.318  0.1876
education12+yrs -1.40321    0.83416  -1.682  0.0925 .
spontaneous     2.04591    0.31016   6.596 4.21e-11 ***
induced         1.28876    0.30146   4.275 1.91e-05 ***
---
```

(Dispersion parameter for poisson family taken to be 1)
Null deviance: 730.253 on 39 degrees of freedom
Residual deviance: 11.694 on 16 degrees of freedom
AIC: 157.56
Number of Fisher Scoring iterations: 18
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 316.17  on 247  degrees of freedom
Residual deviance: 257.80  on 241  degrees of freedom
AIC: 271.8

Number of Fisher Scoring iterations: 4

> model2 <- glm(case ~ parity+education+spontaneous+induced,
+ data = infert, family = binomial())
> summary(model2)

Call:
glm(formula = case ~ parity + education + spontaneous + induced,
  family = binomial(), data = infert)

Deviance Residuals:
  Min       1Q   Median       3Q      Max
-1.8372  -0.8194  -0.4737   0.8909   2.5822

Coefficients:
               Estimate Std. Error z value Pr(>|z|)
(Intercept)   0.2646      0.8669  0.305  0.7602
parity        -0.8043     0.1964 -4.095  4.22e-05 ***
education6-11yrs -1.1494     0.7868 -1.461  0.1441
education12+ yrs -1.6123     0.8185 -1.970  0.0489 *
spontaneous    1.9882      0.3048  6.523  6.90e-11 ***
induced        1.2329      0.2986  4.128  3.66e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 316.17  on 247  degrees of freedom
Residual deviance: 259.43  on 242  degrees of freedom
AIC: 271.43

Number of Fisher Scoring iterations: 4

> pchisq(deviance(model2) - deviance(model1), 1, lower.tail=FALSE)

[1] 0.2019603

Continuing in this manner we find that all the remaining variables are significant at the 5% level
(using the \(\chi^2\) test; you can do this using the drop1 command as well).

5. The dataset \texttt{africa} from the \texttt{faraway} package gives information about the number of military
coups in sub-saharan Africa and various political and geographical information.

Use the AIC to choose a parsimonious generalised linear model for the number of coups. Give an
interpretation of the effect on the response of the variables you include in your model.

\textbf{Solution} Firstly we load the data and remove observations with missing variables. The variable
\texttt{pollib} is converted to a factor.

> library(faraway)
> data(africa)
> africa <- africa[complete.cases(africa),]
> africa$pollib <- factor(africa$pollib, levels=0:2)
It is odd that the number of years since liberation is not included as a variable, but we carry on regardless (see the help function ?africa for details). Fitting an additive model and applying step leaves the variables oligarchy, pollib and parties.

```r
> model1 <- glm(miltcoup ~ ., family=poisson, africa)
> model1a <- step(model1, scope=.

Start: AIC=113.06
miltcoup ~ oligarchy + pollib + parties + pctvote + popn + size + numelec + numregim

Df Deviance AIC
- numelec 1 28.430 111.24
- numregim 1 29.059 111.87
- size 1 29.238 112.05
<none> 28.249 113.06
- pctvote 1 30.572 113.38
- popn 1 30.601 113.41
- oligarchy 1 32.354 115.16
- pollib 2 35.581 116.39
- parties 1 35.311 118.12

Step: AIC=111.24
miltcoup ~ oligarchy + pollib + parties + pctvote + popn + size + numelec + numregim

Df Deviance AIC
- numregim 1 29.081 109.89
- size 1 29.452 110.26
<none> 28.430 111.24
- pctvote 1 30.590 111.40
- popn 1 30.605 111.41
+ numelec 1 28.249 113.06
- pollib 2 36.872 114.68
- parties 1 35.773 116.58
- oligarchy 1 36.595 117.40

Step: AIC=109.89
miltcoup ~ oligarchy + pollib + parties + pctvote + popn + size + numregim

Df Deviance AIC
- size 1 30.040 108.85
- popn 1 30.614 109.42
<none> 29.081 109.89
- pctvote 1 31.599 110.41
+ numregim 1 28.430 111.24
+ numelec 1 29.059 111.87
- pollib 2 37.830 114.64
- parties 1 36.304 115.11
- oligarchy 1 40.291 119.10

Step: AIC=108.85
miltcoup ~ oligarchy + pollib + parties + pctvote + popn

Df Deviance AIC
- popn 1 31.069 107.88
<none> 30.040 108.85
- pctvote 1 32.241 109.05
+ size 1 29.081 109.89
+ numregim 1 29.452 110.26
+ numelec 1 30.002 110.81
- pollib 2 38.022 112.83
- parties 1 37.547 114.36
- oligarchy 1 40.468 117.28

Step: AIC=107.88
miltcoup ~ oligarchy + pollib + parties + pctvote

Df Deviance AIC
- pctvote 1 32.822 107.63
<none> 31.069 107.88
+ popn 1 30.040 108.85
+ size 1 30.614 109.42
+ numregim 1 31.044 109.85
+ numelec 1 31.069 109.88
- parties 1 37.547 112.36
- pollib 2 39.762 112.57
- oligarchy 1 48.196 123.00

Step: AIC=107.63
miltcoup ~ oligarchy + pollib + parties

Df Deviance AIC
<none> 32.822 107.63
+ pctvote 1 31.069 107.88
+ popn 1 32.241 109.05
+ size 1 32.533 109.34
+ numelec 1 32.594 109.40
+ numregim 1 32.643 109.45
- pollib 2 40.025 110.83
- parties 1 38.162 110.97
- oligarchy 1 49.458 122.27

> summary(model1a)

Call:
glm(formula = miltcoup ~ oligarchy + pollib + parties, family = poisson,
data = africa)

Deviance Residuals:
Min 1Q Median 3Q Max
-1.3609 -1.0407 -0.3153 0.6145 1.7536

Coefficients:
Estimate Std. Error z value Pr(>|z|)
(Intercept) 0.207981 0.445679 0.467 0.6407
oligarchy 0.091466 0.022563 4.054 5.04e-05 ***
pollib1 -0.495414 0.475645 -1.042 0.2976
pollib2 -1.112086 0.459492 -2.420 0.0155 *
parties 0.022358 0.009098 2.458 0.0140 *

---
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for poisson family taken to be 1)

Null deviance: 65.945 on 35 degrees of freedom
Residual deviance: 32.822 on 31 degrees of freedom
AIC: 107.63

Number of Fisher Scoring iterations: 5

One can imagine pollib and parties interacting, so we repeat the analysis including this interaction. The interaction is significant, and when it is included a number of other variables become significant.

> model2 <- glm(miltcoup ~ . + pollib:parties, family=poisson, africa)
> model2a <- step(model2, scope="~")
Start:  AIC=107.3
miltcoup ~ oligarchy + pollib + parties + pctvote + popn + size + numelec + numregim + pollib:parties

   Df Deviance  AIC
- numelec 1 18.505 105.31
- numregim 1 19.158 105.97
<none> 1 18.489 107.30
- pctvote 1 20.727 107.53
- popn 1 22.117 108.93
- oligarchy 1 24.907 111.72
- size 1 26.191 113.00
- pollib:parties 2 28.249 113.06

Step:  AIC=105.31
miltcoup ~ oligarchy + pollib + parties + pctvote + popn + size + numregim + pollib:parties

   Df Deviance  AIC
- numregim 1 19.186 104.00
<none> 1 18.505 105.31
- pctvote 1 20.798 105.61
- popn 1 22.429 107.24
+ numelec 1 18.489 107.30
- pollib:parties 2 28.430 111.24
- size 1 26.519 111.33
- oligarchy 1 29.022 113.83

Step:  AIC=104
miltcoup ~ oligarchy + pollib + parties + pctvote + popn + size + pollib:parties

   Df Deviance  AIC
<none> 1 19.186 104.00
- pctvote 1 21.908 104.72
- popn 1 22.435 105.24
+ numregim 1 18.505 105.31
+ numelec 1 19.158 105.97
- size 1 27.030 109.84
- pollib:parties 2 29.081 109.89
- oligarchy 1 33.605 116.41

> summary(model2a)

Call:
  glm(formula = miltcoup ~ oligarchy + pollib + parties + pctvote + popn + size + pollib:parties, family = poisson, data = africa)

Deviance Residuals:
    Min     1Q Median     3Q    Max
-1.1966 -0.7277 -0.1284  0.2610  1.6898

Coefficients:
               Estimate Std. Error z value Pr(>|z|)
(Intercept)  -2.923e+01  1.144e+01  -2.556  0.010595 *
oligarchy     1.083e-01   2.946e-02   3.676  0.000237 ***
pollib1      2.798e+01   1.149e+01   2.434  0.014919 *
pollib2      2.807e+01   1.144e+01   2.453  0.014154 *
parties     2.028e+00    7.657e-01   2.648  0.008088 **
pctvote     1.661e-02   1.010e-02   1.645  0.099977 .
popn        1.392e-02    7.911e-03   1.759  0.078501 .
size      -1.207e-03    4.679e-04  -2.579  0.009895 **
pollib1:parties -1.985e+00   7.701e-01  -2.578  0.009937 **
The final model has positive coefficients for pollib1 and pollib2, which curiously seems to suggest that more liberal countries have more coups. However, to make sense the pollib variable needs to be interpreted together with its interaction with parties. We plot the contribution to the overall rate of coups from these two variables. Note the exponential transform because we used a log link.

```r
> plot(africa$parties, africa$miltcoup, type="n")
> text(africa$parties, africa$miltcoup, africa$pollib)
> x <- 0:62
> y0 <- exp(-29.23 + 2.028*x)
> y1 <- exp(-29.23+27.98 + (2.028-1.985)*x)
> y2 <- exp(-29.23+28.07 + (2.028-2.006)*x)
> lines(x, y0, col="red")
> lines(x, y1, col="blue")
> lines(x, y2, col="green")
```

This plot shows that the strange numbers are the result of the model fitting the cases where pollib is zero rather too closely. It is not plausible that for countries with no liberties the rate of coups
should suddenly skyrocket as soon as you have 13 political parties. The root cause of the problem from the modelling point of view is that we only have two cases where $\text{pollib}$ is zero. Accordingly we combined levels 0 and 1 of $\text{pollib}$ and repeated the analysis.

```r
> x <- africa$pollib == 0
> africa$pollib[x] <- 1
> model3 <- glm(miltcoup ~ . + pollib:parties, family=poisson, africa)
> model3a <- step(model3, scope=".")
```

Start: AIC=114.6

```
miltcoup ~ oligarchy + pollib + parties + pctvote + popn + size + numelec + numregim + pollib:parties
```

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Step: AIC=112.62

```
miltcoup ~ oligarchy + pollib + parties + pctvote + popn + size + numregim + pollib:parties
```

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Step: AIC=110.64

```
miltcoup ~ oligarchy + pollib + parties + pctvote + popn + size + numregim + pollib:parties
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Step: AIC=109.19

```
miltcoup ~ oligarchy + pollib + parties + popn + numregim + pollib:parties
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Step: AIC=108.21

```
miltcoup ~ oligarchy + pollib + parties + popn + numregim + pollib:parties
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Step: AIC=109.19
Step: AIC=108.21
miltcoup ~ oligarchy + pollib + parties + popn + pollib:parties

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Step: AIC=107.92
miltcoup ~ oligarchy + pollib + parties + pollib:parties

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Step: AIC=106.63
miltcoup ~ oligarchy + pollib + parties

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> summary(model3a)

Call:
glm(formula = miltcoup ~ oligarchy + pollib + parties, family = poisson,
data = africa)

Deviance Residuals:
    Min 1Q Median 3Q Max
-1.4012 -1.0593 -0.3945 0.5598 1.7182

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.153867  0.307817 -0.500 0.6172
oligarchy  0.086951  0.021593 4.027 5.65e-05 ***
pollib2 -0.717419  0.285632 -2.512 0.0120 *
parties  0.022562  0.009038 2.496 0.0125 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 65.945 on 35 degrees of freedom

15
Residual deviance: 33.818 on 32 degrees of freedom
AIC: 106.63

Number of Fisher Scoring iterations: 5

We are back where we started! The interaction between pollib and parties was just an artifact of the small number of observations of pollib at level 0. For the final model we see that each year of oligarchy increases the rate of coups by $e^{0.08695} = 1.0908$; full civil rights reduce the rate of coups by $e^{0.7174} = 2.0491$; and each additional political party increases the rate by $e^{0.02256} = 1.0228$. 