Large deviations for supercritical multi-type branching processes

Owen Dafydd Jones

J. Appl. Probab., 41, pp 703–720, 2004

Abstract

Large deviation results are obtained for the normed limit of a supercritical multi-type branching process. Starting from a single individual of type i, let $L[i]$ be the normed limit of the branching process, and let $Z_k^{\min}[i]$ be the minimum possible population size at generation k. If $Z_k^{\min}[i]$ is bounded in k (bounded minimum growth) then we show that $\mathrm{P}(L[i] \leq x) = \mathrm{P}(L[i] = 0) + x^\alpha F^*[i](x) + o(x^\alpha)$ as $x \to 0$. If $Z_k^{\min}[i]$ grows exponentially in k (exponential minimum growth) then we show that $-\log \mathrm{P}(L[i] \leq x) = x^{-\beta/(1-\beta)} G^*[i](x) + o(x^{-\beta/(1-\beta)})$ as $x \to 0$. If the maximum family size is bounded then we get $-\log \mathrm{P}(L[i] > x) = x^{\delta/(\delta-1)} H^*[i](x) + o(x^{\delta/(\delta-1)})$ as $x \to \infty$. Here α, β and δ are constants obtained from combinations of the minimum, maximum and mean growth rates, and F^*, G^* and H^* are multiplicatively periodic functions.