Exploring decision makers’ use of price information in an ‘efficient’ speculative market

J.E.V. Johnson, O.D. Jones, L. Tang

Abstract

We explore the extent to which the decisions of participants in a speculative market effectively account for information contained in prices and price movements. In particular, we investigate how successful they are relatively at incorporating transparent and obscure information. The horserace betting market is chosen as an ideal environment to explore these issues. A conditional logit model is used to determine winning probabilities, based on prices and their movements. The model incorporates techniques for parameterising price (odds) curves and a novel approach is used to test the model against a random alternative. The results challenge the consensus from previous studies that the market is weak form efficient. This disparity arises since previous studies assess the degree to which individual variables are incorporated in final prices, whereas the model we develop uses combinations of variables and their interactions. In addition, we conclude that transparent information, particularly that associated with an enhanced prospect of success, is largely accounted for in the decisions of market participants, but that more obscure information is not properly accounted for.

1. Introduction

There is a large body of evidence which suggests that the combined judgements of decision makers within financial markets effectively incorporate information concerning historical prices into current prices. Consequently, markets are regarded as ‘weak form’ efficient to the extent that abnormal returns cannot be made if buy/sell decisions are made on the basis of historical prices. The horserace betting market is one form of financial market that has received considerable scrutiny in this regard. An important reason for this focus is that “wagering markets are especially simple financial markets, in which the pricing problem is reduced. As a result,
wagering markets can provide a clear view of pricing issues which are complicated elsewhere” (Sauer, 1998, p. 2021). In particular, within horserace betting markets each asset has a definite termination point at which its value becomes clear. In addition, betting markets share many features with wider financial markets including a large number of participants, access to a wealth of information, an uncertain return on investment, and the possibility of ‘insider trading’. Participants in betting markets make decisions concerning the nature and extent of their involvement based on a subjective evaluation of the likely outcome; the outcome in turn affects their welfare. Consequently, analysis of behaviour in these markets can offer important insights into information and decision-making processes in broader environments associated with risk.

The subjective judgements of decision makers within horserace betting markets lead, via their betting (bettors) or odds setting (bookmakers) decisions, to the odds which prevail in the market. The consensus to emerge from previous betting market studies across the world is that the resulting odds fully reflect historical odds information. However, a failing of these studies has been their focus on a single aspect of historical price information. Consequently, the principle approach of this paper is to develop a model for winning probabilities, employing a range of price variables and their interactions, that enables us to make a positive return; thereby demonstrating that the market is not weak form efficient. This will also enable us to explore bettors’ relative success in incorporating transparent and more obscure information.

The paper proceeds as follows. Section 2 provides an overview of the UK horserace betting market, and highlights the degree to which this setting embodies features that are likely to promote market efficiency. Section 3 offers a brief review of the literature that addresses weak form efficiency in betting markets. Section 4 outlines the data employed in this study, provides a rationale for the model which is used to test for market efficiency, and describes how the data is parameterised. The fitted model is presented in Section 5 together with a discussion of its significant components. In Section 6 the model is applied to test data using two different betting
strategies. We show that it is possible to make a profit with both strategies, indicating that the betting market is inefficient. In Section 7 we compare our model with a ‘random’ alternative, to establish that the results produced using the model cannot be attributed to chance. Finally in Section 8 we summarise our conclusions.

2. The UK horserace betting market

There are two distinct forms of horserace betting market that operate in parallel at racetracks in the UK; the bookmaker market, which forms the setting for this study, and the parimutuel market. The odds in the parimutuel market are determined (as in the USA) solely by the pattern of relative bettor demand, according to a pre-determined formula, and bets are settled at the odds prevailing at the close of the market. In contrast, the odds on offer in the bookmaker market are determined by both the decisions of bettors and bookmakers and bets are settled at the odds available in the market at the time the bet is struck. The more serious bettors and those with access to inside information are most likely to bet in the bookmaker market since they can secure their return, without the possibility of a bandwagon effect eroding their gains (which can happen in the parimutuel market).

Independent bookmakers operate at each racetrack and post odds at the commencement of the market before each race. Bettors are then free to bet with the bookmaker offering the best odds on their selection up to the off-time. These odds change according to the relative weight of demand, reflecting bettors’ opinions, and according to each particular bookmaker’s subjective view of the horses’ relative prospects.

In making their wagering decisions bettors may use information from a variety of sources, including fundamental analysis based on the horses’ previous form, advice from specialist publications and information derived from both movements in the bookmaker odds, and the relative amounts of money wagered on horses in the parimutuel system (displayed on computer screens throughout the course of the market). In addition to the on-course market there is a large
(≈90% of total market) off-course market in the UK. Bettors are permitted to bet in betting shops away from the track and are provided with televised coverage and information on the evolving odds at the racetrack. The off-course bookmakers manage their liabilities by placing bets with on-course bookmakers. Consequently, odds in the on-course bookmaker market incorporate information from a wide variety of sources.

The UK bookmaker betting market embodies a number of factors that are likely to lead to a close correspondence between subjective judgements of market decision makers (which emerge as odds) and the relevant objective probabilities of the horses’ success. These include expertise, incentives and feedback (Johnson and Bruce, 2001). Expertise has been shown to aid calibration and odds are determined by a range of individuals with considerable expertise; these include the bookmakers, who devote considerable resources to accurate odds assessment, and insiders, such as owners and trainers, who may be in possession of private information concerning, for example, the horse’s fitness. In addition, bettors clearly have a financial incentive to make accurate judgements and the betting task is a repetitive one, involving prediction, where the stimuli remain relatively constant and where feedback is immediate and regular. All these factors have been shown, in other settings, to improve calibration (Ashton, 1992; McClelland and Bolger, 1994; Wright and Ayton, 1988). In addition, Ceci and Liker (1986) demonstrated that expert bettors employ cognitively complex models involving a range of variables when assessing a horse’s prospects. These observations suggest that decision makers in betting markets are likely to employ information efficiently, resulting in final odds that are a good reflection of each horse’s prospects.

3. Weak form efficiency in horserace betting markets

The overwhelming consensus to emerge from previous horserace betting market studies is that the subjective probabilities, inherent in the odds, closely correspond with the observed win probabilities (Hoerl and Fallin, 1974). In addition, previous studies appear to demonstrate that
historical price information is efficiently incorporated into final odds, to the extent that wagering strategies based on historical prices do not generally yield positive expected returns. Weak form efficiency studies in betting markets fall into three broad categories: those which examine the extent to which horses’ with the lowest/highest odds under/over estimate the horses’ observed win probabilities (the ‘favourite-longshot bias’); those which explore whether the information contained in odds movements is fully discounted in the final odds and those which test whether profitable trading rules can be constructed employing arbitrage between two or more equivalent bets. We briefly discuss each category in turn.

Studies exploring the distribution of odds in horserace betting markets offer strong evidence for a consistent over/under estimation of the probability of longshots/favourites winning (the favourite-longshot bias). These conclusions have been reached for studies widely dispersed in time (McGlothlin, 1956; Ali, 1977; Ziemba and Hausch, 1986; Bruce and Johnson, 2000) and across a variety of countries such as USA (Asch, Malkiel and Quandt, 1982; Snyder, 1978; Thaler and Ziemba, 1988, Ziemba and Hausch, 1986), Australia (Bird and McRae, 1994; Tuckwell, 1983) and the UK (Bruce and Johnson, 2000; Dowie, 1976; Henery, 1985; Vaughan-Williams and Paton, 1997). However, these studies do not generally detect opportunities for trading profitably on this information\(^1\); suggesting that historical prices are efficiently discounted in current odds.

The second set of studies explores the information content associated with changes in odds. Information held by those with privileged information may be transmitted to the market via their betting behaviour; this may be revealed as bookmakers adjust odds to account for their liabilities. In general, previous studies suggest that betting strategies based on these odds adjustments do not yield positive expected returns. Dowie (1976) found, on average, a close correlation between initial and final odds, suggesting little scope for discerning information from odds movements.

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\(^1\) The one exception, a study conducted by Ziemba and Hausch (1986) in the USA, identifies a small expected
However, his analysis failed to discriminate between horses whose odds fell and those whose odds increased throughout the life of the market. Crafts (1985) focussed on horses whose final and initial odds differed significantly. A strategy based on backing/avoiding horses whose odds fell/lengthened significantly produced a better return than a random betting strategy, but it did not produce a positive expected return. This indicates that horses which ease/firm in the market perform worse/better than the original bookmaker odds would imply and Dowie (2003) estimates that approximately 25% more information is incorporated into the final odds (compared with opening odds).

Since bookmakers have a strong incentive, and the commercial opportunity, to be fully appraised of all publicly available information, these observations suggest that betting activity reveals information that is not readily available prior to the formation of the market. This is a view confirmed by Tuckwell (1983), Asch, Malkiel and Quandt (1982) and Schnytzer, Shilony and Thorne (2003), who all demonstrate an increase in the flow of useful information as each betting market evolves.

Taken together, studies exploring odds changes suggest that privately held information is revealed throughout the course of the betting market, but no study finds opportunities for profitable trading once the information becomes available. Sauer (1998, pp. 2048-2049), in an extensive review of the betting market literature, concludes that “the evidence suggests that an informed class of bettors is responsible for altering prices in these markets . . . and the opinions of ‘experts’ appear to be fully discounted in market prices”.

The third set of studies explores weak form efficiency of horserace betting markets by attempting to construct arbitrage betting strategies, employing information revealed within one market in a parallel market. For example, Ali (1977) compares the odds on horses in ‘the daily double’ pool (a parimutuel bet paying a return only if a bettor selects the winners of two races) profit from betting on a very small number of extreme favourites (odds < 3/10).
with the odds in the win pools of the two races. His analysis confirms that the win pool odds provide efficient estimates of the daily double pool odds; a result consistent with the hypothesis that the betting market is weak form efficient. Hausch, Ziemba and Rubinstein (1981) use probabilities revealed by win pool betting to construct probabilities of horses finishing second and third. They then construct profitable wagering strategies based on differences between these probabilities and the odds available in the place and show pools\(^2\). Also, some studies (Hausch and Ziemba, 1990; Schnytzer and Shilony, 1995; Leong and Lim, 1994) demonstrate that positive returns are possible by exploiting the degree to which bets placed in independent win pools (e.g. operated in different locations) on a given race produce differences in odds. However, the number of opportunities for profitable betting employing the above strategies are limited; in some cases the information required to exploit the opportunity is not available in real time, and in others the degree of inefficiency is small. In addition, there is evidence that publicity concerning arbitrage opportunities has lead to a change in betting behaviour which has removed the inefficiency (Ritter, 1994). Consequently, this third set of studies offer little more than a “crease in what is predominantly a smooth pattern of efficiency in the racetrack betting market” (Sauer, 1998).

From the preceding discussion it is clear that horserace betting appears to conform to the expectations developed from calibration studies in other contexts, namely the odds at market close efficiently incorporate information concerning past odds and odds movements. Our concern with this conclusion is that previous tests of weak form efficiency in horserace betting markets have been restricted to the impact of a single variable, such as odds. However, Ceci and Liker (1986) demonstrate that expert horserace handicapping requires bettors to combine

\(^2\) In the parimutuel market, separate pools are created for win bets (those which attempt to select the winner), place bets (those which attempt to select the horse to finish second) and for show bets (those which attempt to select horses to finish second or third). Odds are determined separately in each pool by the relative amount of money on each horse.
relevant information in complex interactive models. We therefore set out to demonstrate that the
decisions of participants in horserace betting markets (as revealed by the odds at market close)
do not efficiently incorporate information concerning a combination of odds variables, including
relevant interactions.

4. Description of the data and type of model

There is evidence that those with access to privileged information (e.g. trainers, owners) chose to
bet with bookmakers rather than in the parimutuel market (Crafts, 1985; Schnytzer and Shilony
1995). Moreover, “the evidence suggests that an informed class of bettors is responsible for
altering prices in these markets” (Sauer, 1998, p. 2049). Accordingly, we collected bookmaker
odds data from 1,200 races run at 41 different racetracks in the UK over the period April – June
1998. Only ‘flat’ races of less than 2 miles were included. The data were supplied by SIS, an
organization that relays on-track bookmaker odds to off-track betting offices. The number of
horses in each race varies from 2 to 20, with a mode of 11. The betting period for each race lasts
from 2.5 to 30 minutes, and averages 12.5 minutes ($\sigma = 4.5$ minutes). The odds for a given horse
in the sample can change from 0 to 10 times, with a mode of 2 and an average of 2.7 changes.

The information employed consists of, for each race $i$ and each horse $j = 1, \ldots, k(i)$ in race $i$, a
sequence of times and odds $\{(t_{ij}(1), u_{ij}(1)), \ldots, (t_{ij}(n), u_{ij}(n))\}$. The final pair $(t_{ij}(n), u_{ij}(n))$ is
always the ‘off time’ and ‘final odds’. The length of the sequence $n = n(i,j)$ varies from horse to
horse, and the number of horses $k = k(i)$ varies from race to race. When we are in the context of a
single race we will drop the index $i$ and when in the context of a single horse we will drop the
indices $i$ and $j$. We scale the times so that $t(1) = 0$ and $t(n) = 1$. We regard the odds as a function
of time, piecewise constant with jumps where the odds change, and call this function a ‘price
curve’. The left-hand diagram of Figure 1 displays a typical example of a price curve: the points
* are the points $(t(1), u(1)), \ldots, (t(n), u(n))$. Note that the final pair $(t(n), u(n))$ are generated by
the start of the race and not by a change in odds, so $u(n) = u(n-1)$.
Taking the price curves as its input, we wish to build a model for \( p_i = (p_i(1), \ldots, p_i(k)) \), the vector of winning probabilities for race \( i \), where \( p_i(j) = \Pr(\text{horse } j \text{ wins race } i) \). Suppose that for horse \( j \) in race \( i \) we have parameterised the price curve by \( x_{i,j} = (x_{i,j}(1), \ldots, x_{i,j}(m)) \), where \( m \) is fixed over all \( i \) and \( j \). We use a conditional logit model for the \( p_i \). That is, for a fixed vector of coefficients \( \beta = (\beta(1), \ldots, \beta(m)) \), we suppose that

\[
p_i(j) = \frac{\exp(<\beta, x_{i,j}>)}{\sum_{l=1}^{k(i)} \exp(<\beta, x_{i,l}>)},
\]

where \(<\beta, x_{i,j}>) = \sum_{h=1}^{m} \beta(h)x_{i,j}(h) \).

We justify this choice of model by noting that it allows the exponent \(<\beta, x_{i,j}>) \) to be interpreted directly as the ability of horse \( j \), independent of the race \( i \). To see this, suppose that \( \varepsilon_i(j), j = 1, \ldots, k(i) \) are independent identically distributed random variables with the double exponential distribution. That is \( \varepsilon_i(j) \) has cumulative distribution function \( F_{\varepsilon_i}(x) = \exp(-\exp(-x)) \), for \(-\infty < x < \infty \). If we put \( W_i(j) = <\beta, x_{i,j}>) + \varepsilon_i(j) \) then it can be shown that \( \Pr(W_i(j) \geq W_i(l), l = 1, \ldots, k(i)) = p_i(j) \) (Maddala, 1983). We can interpret \( W_i(j) \) as a ‘winningness’ index. That is, the winner of race \( i \) is the horse with maximal \( W_i(j) \), and we can interpret the deterministic component \(<\beta, x_{i,j}>) \) of \( W_i(j) \) as a direct measure of horse \( j \)'s ability.

It is possible to use a different distribution for \( \varepsilon_i(j) \), the random component of \( W_i(j) \). For example, using a normal distribution for \( \varepsilon_i(j) \) leads to a multinomial generalization of the probit model. However, this model is computationally more difficult to work with. Moreover, the conditional logit model has been successfully employed by many authors for a range of discrete choice problems, it has been demonstrated to give similar results to probit models (e.g. McFadden, 1974), it allows the number of horses to vary from race to race, it places no importance on the order of the horses and it provides a mechanism for expressing the competition between horses.
If we observe \( N \) races, and the winner of race \( i \) is horse \( j^* \), then the joint likelihood \( L = L(\beta) \) is the probability of observing this set of results, assuming the \( p_i \) are as above. That is

\[
L(\beta) = \prod_{i=1}^{N} p_i(j^*) = \prod_{i=1}^{N} \frac{\exp(<\beta, x_{i,j^*}>)}{\sum_{l=1}^{t(i)} \exp(<\beta, x_{i,l}>)}.
\] (2)

We employ maximum likelihood estimation, as implemented in Limdep (Green, 1998), to choose \( \beta \) that maximizes \( L(\beta) \).

4.1. Parameterising the price curve

To construct a model for \( p_i(j) \) we require a consistent parameterisation of the price curve for each horse. In doing so we have two aims: to provide a general summary of the shape and other physical characteristics of the price curve, and to pick out particular features that have been identified in the literature or by active gambles as having an effect on the horse’s winning probability. For all the parameters we consider there is an implicit dependence on the race \( i \) and horse \( j \), though, to simplify, we will not make this explicit in the notation.

4.1.1. Orthogonal polynomial expansion

We are interested to include parameters that reflect the general shape of the price curve, since the precise shape of a price curve that is likely to signal a potential winner (or loser) is unclear. To provide a general summary of the shape of a price curve \{\( (t(1), u(1)), \ldots, (t(n), u(n)) \)\}, we use an orthogonal polynomial expansion of order 3. This allows us to measure the height of the curve (final odds), the linear trend, the curvature (quadratic component) and change in curvature (cubic component). By using an orthogonal polynomial expansion, we can measure the size of each component (constant, linear, quadratic and cubic) independently of the others. Orthogonal polynomials are a classical statistical tool; details of their construction can be found for example in Wetherill (1981). We summarise the procedure here and it is illustrated in Figure 1.
Figure 1. Orthogonal polynomial decomposition of a price curve. The first diagram shows the price curve and its constant, linear, quadratic and cubic approximations. The second diagram shows the separate constant, linear, quadratic and cubic components, which are added to give the approximations in the first diagram.

Given a set of points \(\{(t(1),u(1)), \ldots, (t(n),u(n))\}\), an orthogonal polynomial basis is a sequence of polynomials \(f_0, f_1, f_2, \ldots\) such that \(f_i\) is of order \(i\) and

\[
\sum_{i=1}^{n} f_i(t(l)) f_j(t(l)) = 0 \quad \text{for all } i \neq j.
\]  

(3)

It can be shown that an orthogonal polynomial basis always exists and that there is a unique set of coefficients \(a_0, a_1, a_2, \ldots\) such that

\[
u(l) = \sum_{i=0}^{\infty} a_i f_i(t(l)) \quad \text{for all } l = 1, \ldots, n.
\]

(4)

The \(a_i\) can be found by least squares. By restricting ourselves to an order 3 expansion we get an approximation to \(u\). As the \(f_i\) are orthogonal, we can interpret \(a_i\) as the size of the order \(i\) component in the price curve. In fact, the equations (3) do not specify a unique basis, and we can impose further constraints without compromising orthogonality. In our case, because of the importance of the final odds \(u(n)\), we take \(f_0(t(n)) = f_0(1) = 1\) and \(f_i(1) = 0\) for all \(i \geq 1\). The effect of this is to make the constant component \(a_0\) equal to \(u(n)\). We also norm each \(f_i\) so that its leading term is simply \(t^i\). In particular this implies that \(a_1\) is the slope of the least squares regression line constrained to pass through \((t(n),u(n))\). We interpret \(a_2\) as a measure of the curvature of the price curve and \(a_3\) as a measure of the change in curvature.
A potential problem with polynomial expansions is that they are unstable when only a small number of points are used. That is, if \( n \) is small, then a small change in one of the \( (t(i), u(i)) \) can produce a large change in \( a_2 \) and \( a_3 \). To mitigate this, we regularise the procedure by introducing a roughness penalty when fitting the \( a_i \). Let \( F(t) = \sum_{i=0}^{n} a_i f_i(t) \), then we choose the \( a_i \) to minimize

\[
\sum_{i=1}^{n} (u(i) - F(t(i)))^2 + \lambda G(F) \tag{5}
\]

where \( G(F) \) is the roughness penalty and \( \lambda \) is some constant of proportionality. Typically \( G(F) \) is some measure of curvature such as a Sobelov norm (that is, a norm based on first, second and sometimes higher order derivatives of \( F \)). However, the slope of \( F \) at \( t(n) \) is of particular interest to us as a measure of late price movement (see below), so we do not want to depress this unnecessarily. So, instead of a Sobelov norm, we put \( G(F) \) equal to the area of \( F \) above \( u_{\text{max}} = \max u(i) \) and below \( u_{\text{min}} = \min u(i) \). That is,

\[
G(F) = \int_0^1 \max(F(t) - u_{\text{max}}, 0) dt + \int_0^1 \max(u_{\text{min}} - F(t), 0) dt . \tag{6}
\]

4.1.2. Volatility measures

The volatility or roughness of a curve is one of its fundamental characteristics. Two parameters were used to measure the volatility of the price curve, the number of price changes and the absolute variation. That is, for price curve \( \{(t(1), u(1)), \ldots, (t(n), u(n))\} \), we put

\[
b_1 = n - 2 \quad \text{and} \quad b_2 = \sum_{i=2}^{n} |u(i) - u(i-1)| . \tag{7}
\]

Clearly there will be some degree of correlation between \( b_1 \) and \( b_2 \). There are three specific types of behaviour that the volatility can capture, which would not be revealed by the orthogonal expansion alone. The first is the ‘bandwagon effect’ (Schnytzer and Shilony, 2003; Smith, 2003), whereby an odds reduction prompts further betting, which triggers a further odds reduction, and
so on. The second is the slow drift of odds for horses that are not backed (Schnytzer and Shilony, 2003). This occurs because bookmakers initially depress odds, but then allow them to improve gradually if a horse is not backed. The third relates to the disclosure of significant, perhaps contradictory pieces of information concerning the horse’s prospects as the market evolves (see Section 5.1).

4.1.3. More specific features

The remaining three parameters were included since previous literature suggests that they may have some influence on a horse’s probability of success.

A number of studies, as indicated in Section 3, have demonstrated a close correspondence between probabilities implied by final odds and winning probabilities (e.g. Bruce and Johnson, 2000). The final odds for horse $j$ in race $i$ imply a probability of winning $q_j$, via the relationship

$$u_{i,j}(n) = \frac{1 - q_j(j)}{q_j(j)}, \quad q_j(j) = \frac{1}{1 + u_{i,j}(n)}.$$

(8)

We call this the ‘track probability’ to distinguish it from the model probability $p_i(j)$. We already include the final odds in the model as $a_0 = u_{i,j}$, which gives $p_i(j) \propto \exp(\beta (a_0) \cdot (1 - q_j(j))/q_j(j))$. However, it is plausible that a more direct relationship between $p_i(j)$ and $q_j(j)$ would result in a better fit. Accordingly, we include the parameter $c_1 = \log q_j(j) = -\log (1 + u_{i,j}(n))$, which gives $p_i(j) \propto \exp(\beta (c_1) \cdot c_1) = q_j(j)^{\beta (c_1)}$. In fact, Chapman (1994) even found that $c_1$ added significant explanatory power in a sophisticated fundamental handicapping model that included 20 variables associated with the horse and its jockey.

Crafts (1985), Tuckwell (1983) and Bird and McCrae (1987), amongst others, have demonstrated that a horse’s enhanced prospects of success are revealed by a large reduction in odds from the start to the completion of the market. Consequently, we take $c_2 = \text{final odds} - \text{initial odds} = u_{i,j}(n) - u_{i,j}(1)$, although this will be highly correlated with the slope $a_1$. 
Those with access to privileged information have an incentive to bet late in the market. This allows them to capitalise on the general drifting out of odds as the market progresses, confirm that their horse gets to the start without mishap and it enables them to become more fully informed about rival horses’ prospects, via odds changes (Asch, Malkiel and Quandt, 1983; Schnyter, Shilony and Thorne, 2003). Consequently, we seek to include information in our model concerning late changes to odds. Let \( u(t) \) be the price curve, and \([a, b]\) be a small subinterval of \([0, 1]\), close to 1. We take as our measure of late change the most extreme slope \((u(1) - u(t))/(1 - t)\) for \( t \in [a, b]\), that is, the slope with the largest absolute value. We let the late change parameter be \( c_3 \), and provide two illustrations in Figure 2, where \( c_3 \) is the slope of the line plotted though \((1, u(1))\).

\[
\mathcal{F}(t) = a_1 + 2a_2 + 3a_3.
\]

Values of \(a\) and \(b\) were chosen to make \( c_3 \) reasonably robust, so that small changes in \( u(t) \) do not produce large changes in \( c_3 \), while minimising correlation with the overall trend \( a_1 \). Taking \([a, b] = [0.9, 0.95]\) gave reasonable results.

If the price curve were smooth, then \( c_3 \) would simply be an approximation to the derivative at \( t = 1 \). Consider again our orthogonal polynomial expansion of the price curve, \( F(t) = \sum_{i=0}^{3} a_i f_i(t) \). \( F(t) \) is a smooth approximation of the price curve, so we expect \( c_3 \) to be highly correlated with \( F'(1) = a_1 + 2a_2 + 3a_3 \).

4.1.4. Rescaling and splitting
Two refinements of the parameter set were incorporated, based on our understanding of how prices behave in practice. Firstly, it is known that on long-odds horses (those with high final odds), the odds change by larger amounts than for short-odds horses, and we believe that the relative size is more important than the absolute size of any change. Consequently, before calculating parameters \( a_1, a_2, a_3, b_2, c_2 \) and \( c_3 \), the price curve \{\( (t(1),u(1)), \ldots, (t(n),u(n)) \)\} was rescaled by dividing \( u(i) \) by \( u(n) \) for \( i = 1, \ldots, n \). Parameters \( a_0 \) and \( c_1 \), which are based on the final odds, were not rescaled, and \( b_1 \) is unaffected by this rescaling. Secondly, practicing gamblers interpret the price curve differently when the odds are coming in (decreasing) or going out (increasing). This suggests that we should interpret price changes differently when they are changes down rather than up. Accordingly, all parameters except \( a_0 \) and \( c_1 \) were split into two parts, \( x^+_i \) and \( x^-_i \), depending on whether the odds came in or went out. That is, \( x^+_i = x_i \) if \( u(n) > u(1) \) or 0 otherwise, and \( x^-_i = x_i \) if \( u(n) \leq u(1) \) or 0 otherwise.

5. Model fitting

In order to fit and test the model given in Equation (1) the data set was split into two parts. The first 800 races were used to fit the model, and the remaining 400 used to test it. A stepwise fitting procedure was used to select a set of parameters significant at the 95% level. Pairwise interactions of all the parameters were also considered. In the final model the parameters \( a^+_1, b^-_2, c_1, c^-_3 \) and the interaction \( a^+_1 b_1 \) were all significant at the 95% level. The estimated coefficients \( \beta \) are given in Table 1. The log likelihood ratio of the model over the constant alternative is 857.0468, which gives us that the model is significant with a p-value of 0.0000.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^+_1 )</td>
<td>slope up</td>
<td>-2.0493</td>
<td>0.8110</td>
<td>0.0115</td>
</tr>
<tr>
<td>( b^-_2 )</td>
<td>absolute variation down</td>
<td>-0.4227</td>
<td>0.1907</td>
<td>0.0266</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>log(track probability)</td>
<td>1.1678</td>
<td>0.0648</td>
<td>0.0000</td>
</tr>
<tr>
<td>( c^-_3 )</td>
<td>late change down</td>
<td>-0.0666</td>
<td>0.0331</td>
<td>0.0440</td>
</tr>
</tbody>
</table>

Table 1. Estimated coefficients of the model. Note that \( a^+_1, b^-_2, c^-_3 \) and \( a^+_1 b_1 \geq 0; c_1 < 0 \).
5.1 Interpretation of fitted parameters

Firstly we note that, as $a_1$, $b_2$ and $c_1$ are in the model, it is not surprising that $c_2$, $b_1$ and $a_0$ are not, as we knew these parameters were correlated. We interpret this as saying that $a_1$, $b_2$ and $c_1$, respectively, capture more relevant information concerning overall change in odds, volatility and final odds than $c_2$, $b_1$ and $a_0$. Secondly, we note that some degree of correlation between $c_3$ and $a_1 + 2a_2 + 3a_3$ was expected, so the presence of $c_3$ in the model in part explains why $a_2$ and $a_3$ do not appear.

5.1.1. Track probability and favourite – long shot bias

The parameter $c_1 = \log q_i(j)$, which has coefficient 1.1678, has a large influence. This is because of the relative size of $c_1$ compared to the other parameters. Considering just the effect of $c_1$ on the model probability we have $p_i(j) \propto \exp(1.1678 \log q_i(j)) = q_i(j)^{1.1678}$. We interpret this relationship as a reflection of the so-called favourite-long shot bias. That is, for horses with long odds, the true probability of winning is significantly less than the implied track probability, while for horses with short odds, the true probability of winning is close to the implied track probability. A plot of $\log u_{i,j}(n) = \log (1-q_i(j))/q_i(j)$ against $\log q_i(j)^{1.1678}$ gives a very close match to the analogous plot given in Bruce and Johnson (2000), which was obtained by modelling the favourite-longshot bias directly.

5.1.2. Slope

The parameter $a_1^+$, which has a coefficient significantly different to zero in our model, should be considered in conjunction with the interaction term $a_1^+ b_1$. $a_1^+$ is non-zero and positive when the odds worsen (ie. the least squares regression line constrained to pass through $t(n)$, $u(n)$ has a positive slope). It is reasonable that a horse whose odds move out should be less likely to win since this may reflect lack of confidence in the horse’s prospects on the part of bettors and/or
negative information concerning the horse’s chances being released to the market (Crafts, 1985). The effect of the $a_1^+b_1$ interaction is to reduce this effect when the odds worsen in a large number of small steps, as opposed to small number of large steps. Again this is expected since such a pattern will emerge as bookmakers adjust odds outwards in small steps in order to encourage bets on horses where demand for bets is weak. This contrasts with horses whose odds move out in large steps, as particular pieces of adverse information concerning the horse reach the market (e.g. horse sweating badly in the paddock). It appears that bookmakers’ incremental outward adjustments to stimulate demand are less of a negative signal concerning a horse’s prospects than sharp outward adjustments in odds, which may reflect new adverse information. When the odds come in on a horse then $a_1^+$ is zero, so neither $a_1^+$ nor $a_1^+b_1$ have an effect, but if the odds come in on horse $j$, then they will usually go out on other horses; the combined effect will be to increase $p_i(j)$.

5.1.3. Late change down

The late change down, $c_3^-$ acts as expected. $c_3^-$ is always negative in sign, so when there is a late change down the effect is to increase the probability of winning. This agrees with the belief that a late change down indicates large bets close to off-time by those with access to privileged information. We note that a late change up has no significant effect on the probability of winning. This is probably because at a late stage in the market a significant increase in odds is only likely to arise from new but widely available information, and hence is readily discounted in final odds (e.g. horse bolts on way to start).

5.1.4. Volatility

Absolute variation $b_2^-$ was used as a volatility measure. The coefficient is negative, indicating that high volatility in the price curve makes a horse less likely to win, but only when the odds have come in. That is, inconsistency in the direction of odds movements for horses whose odds fall overall during the betting period is a negative signal concerning the horse’s prospects. Such
behaviour can arise from contradictory indications of the horse’s relative prospects filtering into the market at different times.

5.1.5. Transparent and Opaque Information

The results suggest that transparent information is more efficiently discounted in betting markets than more opaque information. Each of the parameters with significant coefficients might be described as opaque compared with an equivalent, but more transparent parameter excluded from the model; for example, $a_1$ vs. $c_2$, slope of the least squares regression line through $t(n)$, $u(n)$ vs. final - initial odds; $b_2$ vs. $b_1$, the absolute value of (scaled) odds changes vs. number of odds changes; $c_1$ vs. $a_0$, log of the probability implied by track odds vs. track odds. Whilst the late change parameter $c_3$, which appears in the model, has no directly comparable transparent alternative, we found that the coefficient for the scaled version of $c_3$ is significant whereas that for the non-scaled (more transparent) version of $c_3$ is not.

6. Model Testing

Races 801 to 1200, run during May/June 1998, were used to test the model. As we do not have repeated observations (each race is only run once), we must use indirect methods to measure the accuracy of the model. We consider two betting strategies, maximum expected payoff and maximum expected log payoff. For each strategy we use the model probabilities and final odds as inputs, and analyse the returns produced. Given correct probabilities as inputs (as opposed to estimated probabilities), both strategies give non-negative expected returns. Thus, if they give non-negative returns using our model probabilities $p_i(j)$ as inputs, we take this as evidence that the $p_i(j)$ are reasonably accurate. Moreover a positive return indicates that the market is weak form inefficient; since abnormal returns can be made by simply employing historical price information.

6.1 Maximum expected payoff
For race $i$, let $r_i = (r_i(1), \ldots, r_i(k))$ be the returns, that is $r_i(j) = 1 + u_{i,j}(n) = 1/q(j)$ is the amount returned on a unit bet on horse $j$ if the horse wins. Let $b_i = (b_i(1), \ldots, b_i(k))$ give the amount bet on each horse in race $i$. As before, we will drop the subscript $i$ when the context is clear. Under the model, the expected payoff on the race is

$$E(b) = \left( \sum_{j=1}^{k} p(j)b(j)r(j) \right) - \sum_{j=1}^{k} b(j) = \sum_{j=1}^{k} b(j)(p(j)r(j) - 1).$$

(9)

If $p(j)r(j) > 1$ for some $j$, then $E(b)$ is unbounded unless we bound $b$. Accordingly, maximising $E$ subject to the constraint $\sum_{j=1}^{k} b(j) \leq 1$, $b(j) \geq 0$ for all $j$, gives the following strategy. Let $j^*$ be the value of $j$ that maximises $p(j)r(j)$, then $b(j) = 0$ for $j \neq j^*$, and $b(j^*) = 1$ if $p(j^*)r(j^*) > 1$ or 0 otherwise.

Following this strategy from race to race, the total wealth behaves like a random walk with a linear trend (drift). As such, total wealth can grow at most linearly, and there is a non-zero probability of ruin. In practice we modify the strategy in two ways to reduce its variance, and thus reduce the chance of ruin. Firstly, when a bet is indicated, only an amount $b(j^*) = 1/r(j^*)$ is bet (bet to win amount 1). This reduces your exposure on horses that are less likely to win. Secondly, in addition to requiring $p(j)r(j) > 1$, we apply a filter rule, further restricting bets to horses where track probability $q(j) \geq 0.2$, that is $1/r(j) \geq 0.2$. This restriction is chosen since previous studies (e.g. Tuckwell, 1983) indicate that the expected loss to the bettor increases significantly for horses with track probability less than 0.2.

This strategy provides two complementary instructions: which races to bet on, and how much to bet on each horse (at most one per race in this case). Applying this strategy to the test data set, we obtained the following results:

- average profit per race = 0.0056;
- proportion of races bet on = 32%;
- average profit per race bet on = 0.0175;
average profit per pound bet = 0.0469.

Figure 3 plots the cumulative profit under this strategy. The cumulative profit over the test period was 2.2390 (betting up to 1 unit each time), though we note that it did drop to -1.9303 during the test period. As expected, this strategy gives a (positive) linear growth for total wealth, and compares very favourably with a random strategy of either betting an equal amount on each horse (average return -31.59% per race) or betting an amount proportional to the track probability on each horse (average return -17.72% per race).

6.2. Kelly strategy: maximum expected log payoff

In this setting, instead of betting an amount \( b_i(j) \) on horse \( j \) in race \( i \), we bet a fraction \( f_i(j) \) of our current wealth. Let \( \mathbf{f} = (f_1(1), \ldots, f_i(k)) \). As usual, we will drop the subscript \( i \) when the context makes it unnecessary. Betting fraction \( \mathbf{f} \), if horse \( x \) wins then our current wealth will increase by a factor of \( 1 - \sum_{j=1}^{k} f(j) + f(x)r(x) \). The Kelly strategy consists of choosing \( \mathbf{f} \) to maximise the expected log payoff, \( F(\mathbf{f}) \) where

\[
F(\mathbf{f}) = \sum_{x=1}^{k} p(x) \log\left( f(x)r(x) + 1 - \sum_{j=1}^{k} f(j) \right)
\]  
\phantom{=} \quad (10)
This betting strategy was introduced by Kelly (1956). It was later shown to be asymptotically optimal by Breiman (1961), in the sense that it maximises the asymptotic rate of growth for wealth, with 0 probability of ruin. Using the Kelly criterion, the total wealth grows at an exponential rate, though the standard deviation remains proportional to total wealth and thus also grows exponentially. We also note that this strategy only gives 0 probability of ruin if arbitrarily small bets are allowed. In practice this caveat has lead some authors to consider modified Kelly strategies (e.g. Benter, 1994; Ziemba and Hausch, 1986), whereby some fixed fraction of $f$ is bet. As we are interested in the theoretical rather than practical performance of our model, we restrict ourselves to the usual form.

As for the maximum expected payoff strategy, the Kelly strategy tells us which races to bet on, as well as how much to bet on each horse. In this case we can bet on more than one horse in a race, though our bets are restricted to horses that give a positive expected return. Applying this strategy to the test data set, we obtained the following results:

- average profit per race = 0.0032 * wealth;
- proportion of races bet on = 42%;
- average profit per race bet on = 0.0076 * wealth;
- average profit per pound bet = 0.1680.

Figure 4 plots the natural logarithm of the cumulative wealth under this strategy, starting with initial wealth 1. Over the out of sample test period, total wealth increased exponentially by a factor of 2.4597. Thus, although the expected profit per race is significantly less than that obtained using the maximum expected payoff strategy, 0.0076 compared to 0.0175\(^3\), in the long run the Kelly strategy will perform better. This is because the Kelly strategy is cumulative: the size of bets made is proportional to the current wealth, whereas the size of bets made under the maximum expected payoff strategy is fixed. We also observe that the Kelly criterion was more

\(^3\) For the purpose of comparison we take current wealth as 1 when applying the Kelly strategy.
consistent than the maximum expected payoff strategy over the test data set. We suggest that this is because the Kelly criterion effectively spreads the risk more, by betting on more than one horse per race.

Figure 4. Log of cumulative wealth using the Kelly strategy. On the left we have the wealth for races 1 to 800 (those used to fit the model), and on the right the wealth for races 801 to 1200 (the test data set). Here wealth is given as a multiple of original wealth.

7. Comparison with random betting

Both betting strategies considered in the previous section gave a positive return over the test period. To be confident that we can ascribe this to the accuracy of our model, rather than just good luck, we consider how our model compares to an uninformed ‘random’ alternative. That is, we test the hypothesis that our model performs no better than a model that makes no use of the information contained in the price curves, other than the final odds.

For a given race \(i\), we construct a random model \(q_i^* = (q_i^*(1), \ldots, q_i^*(k))\) for the vector of winning probabilities as follows. Let \(r_i = (r_i(1), \ldots, r_i(k))\) be the returns and \(q_i = (q_i(1), \ldots, q_i(k))\) the track probabilities, \(q_i(j) = \exp(-\log(r_i(j)))\). The random model should be a random perturbation of \(q_i\). Accordingly, let \(E_i = (E_i(1), \ldots, E_i(k))\) be a vector of independent normal random variables, mean 0 and variance \(\sigma^2\), and define the random model by
\[ q_i^* (j) \propto \exp(-\log(r_i(j)) + E_i(j)) \] That is, putting \( Z_i(j) = \exp(E_i(j)) \) (so \( Z_i(j) \) has a log-normal distribution),

\[
q_i^* (j) = \frac{\exp(-\log(r_i(j)) + E_i(j))}{\sum_{l=1}^{k} \exp(-\log(r_i(l)) + E_i(l))} = \frac{q_i(j)Z_i(j)}{\sum_{l=1}^{k} q_i(l)Z_i(l)}.
\] (11)

We introduce the random perturbations in this manner to mimic the structure of the conditional logit model. That is, we replace \(<\beta, x_{ij}>\) by \(-\log r_i(j) + E_i(j)\), noting that \(-\log r_i(j) = c_1\) is an important component of \(x_{ij}\). This means that the perturbation to \(q_i(j)\) is multiplicative rather than additive.

In general \(\sum_{j=1}^{k} q_i(j) > 1\), since bookmakers add a margin to their odds. We norm \(q_i = (q_i(1), ..., q_i(k))\) so that it sums to 1 and call the normed track probabilities \(\bar{q}_i = (\bar{q}_i(1), ..., \bar{q}_i(k))\), where \(\bar{q}_i(j) = q_i(j) / \sum_{l=1}^{k} q_i(l)\). To see that \(q_i^*\) has the right distribution, we look at the distribution of \(p_i(j) - \bar{q}_i(j)\) over the test data set, and compare this with the distribution of \(q_i^*(j) - \bar{q}_i(j)\). We choose \(\sigma\) so that the two distributions have the same standard deviation, giving \(\sigma = 1.4\). Figure 5 plots sample histograms for the two distributions. The distribution of \(p_i(j) - \bar{q}_i(j)\) is exact (given the test data set). The distribution of \(q_i^*(j) - \bar{q}_i(j)\) is a Monte-Carlo estimate. We see that while

![Figure 5. Histogram of \(p_i(j) - \bar{q}_i(j)\) (left) and \(q_i^*(j) - \bar{q}_i(j)\) (right), using \(\sigma = 1.4\).](image-url)
the random model does not give an exact fit to the desired distribution, it does capture some of the important features of the shape. Further support for this conclusion is obtained from a QQPlot of the two distributions, which indicates a remarkably good degree of agreement.⁴

We compare the performance of the random model with the conditional logit model in the context of our two betting strategies. In each case we perform a number of Monte-Carlo simulations to estimate the distribution of the average profit per pound per race.

### 7.1. Maximum expected payoff

The random model was employed as follows. For each race \( i \) in the test data set we simulated a set of random model probabilities \( q_i^* \), and then applied the maximum expected payoff strategy to determine how much to bet on each horse. This gives a simulated profit for each race, which we then average over all 400 races in the test data set to get an estimate of the average profit per race \( X \). This procedure is then repeated 1000 times to get an estimate of the distribution of \( X \). Figure 6 gives the empirical cumulative distribution function (empirical CDF) of \( X \).

![Empirical CDF](image)

**Figure 6.** Empirical cumulative distribution function of the average profit per race over the test period for the random model, using the strategy of Maximum Expected Return on the left, and the Kelly Strategy on the right.

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⁴ A QQPlot plots the quantiles of one distribution against those of another. A straight line indicates a good
Using our conditional logit model, the achieved average profit per race was 0.0056. Using the empirical CDF, we estimate \( \Pr(X > 0.0056) = 0.169 \). That is, under the hypothesis that our conditional logit model performs no better than the random model, the probability of achieving an average profit per race of 0.0056 or better is 0.169. In other words, the test has p-value 0.169, which is not particularly significant.

Note that we compared the profit per race rather than per race bet on, as the decision whether or not to bet depends on the probabilities used. Using the random model we bet on 24% of races on average (cf. 32% using our conditional logit model). To judge the sensitivity of the test to the value of \( \sigma \), we repeated the test for a number of different values of \( \sigma \). The results are given in Table 2.

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
<th>1.4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E(X) )</td>
<td>-0.00071</td>
<td>-0.0042</td>
<td>-0.0045</td>
<td>-0.0040</td>
<td>-0.0043</td>
<td>-0.0037</td>
</tr>
<tr>
<td>( \Pr(X &gt; 0.0056) )</td>
<td>0.124</td>
<td>0.170</td>
<td>0.180</td>
<td>0.169</td>
<td>0.177</td>
<td>0.181</td>
</tr>
</tbody>
</table>

We see that the test is insensitive to the value of \( \sigma \) used. The relatively high probability that the random model results in a positive profit over the test period can be attributed to the favourite-long shot bias. Our betting strategy is restricted to horses with track probability \( q_i(j) > 0.2 \) (odds of 4/1 or lower). As has been previously noted, the track probability tends to be an underestimate for short odds horses, meaning that one has a greater chance of a positive return by betting on short odds horses.

We also note that 100% of the time, this random strategy performed better than the naïve strategies of betting an equal amount on each horse, or betting to win 1 pound on each horse.

### 7.2. Kelly strategy: maximum expected log payoff
For each race $i$ in the test data set we generated a set of random probabilities $q_i^*$, to which we applied the Kelly strategy to determine how much to bet on each horse. This was repeated for all 400 races in the test data set to obtain an average profit per pound per race, denoted $Y$. This whole procedure was then repeated 500 times to obtain an estimate of the distribution of $Y$. The empirical CDF of $Y$ is given in Figure 6.

Using our conditional logit model, the achieved average profit per pound per race was 0.0032. Using the empirical CDF we estimate $\Pr(X > 0.0032) = 0.006$. That is, under the hypothesis that our conditional logit model performs no better than the random model, the probability of achieving an average profit per pound per race of 0.0032 or better is 0.006. In other words, the test has p-value 0.006, which is highly significant.

To judge the sensitivity of the test to the value of $\sigma$, we repeated the test for a number of different values of $\sigma$. The results are given in Table 3.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>1</th>
<th>1.4</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(X)$</td>
<td>-0.00037</td>
<td>-0.0043</td>
<td>-0.0360</td>
<td>-0.1037</td>
<td>-0.1222</td>
<td>-0.1747</td>
</tr>
<tr>
<td>$\Pr(X &gt; 0.0032)$</td>
<td>0.000</td>
<td>0.006</td>
<td>0.000</td>
<td>0.000</td>
<td>0.006</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Again, the test is insensitive to the value of $\sigma$ used. Given such low p-values, we can be confident in saying that the conditional logit model is not assigning probabilities at random. The reason that this is brought out when using the Kelly strategy as opposed to the maximum expected payoff strategy, is that the Kelly strategy requires good estimates of the win probability for every horse in a race if it is to be successful. In other words, because the Kelly strategy uses $p_i$ more extensively than the maximum expected payoff strategy, it is more sensitive to errors in $p_i$. Using the Kelly strategy, the amount bet on horse $j$ depends on $p_i(j)$ for each $j$. Using the maximum expected payoff strategy, only the $p_i(j)$ for which $p_i(j)r_i(j)$ is largest is important, and $p_i(j)$ is only used to decide which horse to bet on, not how much to bet.
8. Conclusions

In this paper we set out to explore whether the bookmaker horserace betting market fully incorporates a variety of historical price information variables, including interaction effects. We conclude that it does not; suggesting that the market is weak form inefficient. This conclusion is in sharp contrast to the majority of studies examining betting market efficiency. We believe this is because previous studies have focussed on assessing the extent to which individual factors are efficiently incorporated into prices, rather than looking at combinations of a range of variables.

In addition, previous studies have utilised readily discernable variables, most of which signal a horse’s enhanced prospects of success.

Our findings also suggest that market participants are largely effective in discounting transparent information concerning a horse’s prospects in their decisions but they do not appear to incorporate more obscure information: odds and the manner in which odds move, are rich but subtle information sources, which bettors do not fully utilize.

In summary, this paper adds to our knowledge of the degree to which different types of information are discounted in decisions made in betting markets. It also introduces techniques for parameterising price curves and a novel approach for testing a model for producing probabilities, by comparison against a random alternative. Future work exploring other financial markets, using the techniques introduced here, may yield interesting conclusions regarding market efficiency and the manner in which information is employed by market participants.

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