1) Let $T: V \to V$ be a self-adjoint linear operator and let $v$ be an eigenvector of eigenvalue $\lambda$. Show that $\lambda \in \mathbb{R}$.

2) Let $T: V \to V$ be a self-adjoint linear operator. Let $\lambda$ and $\gamma$ be eigenvalues of $T$ with $\lambda \neq \gamma$. Let $X_\lambda = \{ v \in V | Tv = \lambda v \}$ and $X_\gamma = \{ v \in V | Tv = \gamma v \}$. Show that $X_\lambda$ is orthogonal to $X_\gamma$.

3) Let $H$ be a Hilbert space and let $T: H \to H$ be a compact linear operator. Let $\lambda$ be a non-zero eigenvalue of $T$ and let $X_\lambda = \{ v \in H | Tv = \lambda v \}$. Show that $X_\lambda$ is finite-dimensional.

4) Let $T: V \to V$ be a bounded linear operator.
   (a) Show that $T^*T$ is self-adjoint.
   (b) Show that if $\gamma$ is an eigenvalue of $T^*T$ then $\gamma \in \mathbb{R}_{\geq 0}$.