Due 5pm on 4 September in the appropriate assignment box on the ground floor of Richard Berry.

1. Define the following and give an example for each:
   (a) cardinality,
   (b) finite,
   (c) infinite,
   (d) countable,
   (e) uncountable.

2. Prove that Card(\(\mathbb{Z}_{>0}\)) \(\neq\) Card(\(\mathbb{R}\)).

3. Define the following and give an example for each:
   (a) sequence,
   (b) converges (for a sequence),
   (c) diverges (for a sequence),
   (d) limit (of a sequence),
   (e) sup (of a sequence),
   (f) inf (of a sequence),
   (g) lim sup (of a sequence),
   (h) lim inf (of a sequence),
   (i) bounded (for a sequence),
   (j) increasing (for a sequence),
   (k) decreasing (for a sequence),
   (l) monotone (for a sequence),
   (m) Cauchy sequence.

4. Give an example of a sequence \((a_n)\) such that none of \(\inf a_n\), \(\liminf a_n\), \(\limsup a_n\), and \(\sup a_n\) are equal.

5. Find the power series expansions and the radius of convergence of \(e^x\), \(\log(1 + x)\), \(\frac{1}{1-x}\), \((1 + x)^{1/2}\), arctan \(x\), and sinh \(x\).

6. Let \(r \in \mathbb{R}\) with \(0 < r < 1\). Find \(\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n\), and explain why this limit is important to everyone with a credit card.
7. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges and that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

8. Let $r \in \mathbb{R}$. Find (with proof) $\sum_{n=1}^{\infty} r^n$.

9. Show that the alternating harmonic series for $\arctan 1$ is conditionally convergent but not absolutely convergent. Explain how to rearrange it so that its sum is 301.