1. Classify and construct the finite dimensional simple modules for $U_q\mathfrak{sl}_2$, where $U_q\mathfrak{sl}_2$ is the algebra generated by $E, F, K^\pm 1$, with relations

$$KEK^{-1} = q^2 E, \quad KFK^{-1} = q^{-2} F, \quad \text{and} \quad EF - FE = \frac{K - K^{-1}}{q - q^{-1}}.$$ 

2. Define the symmetric group (via permutations).

3. Show that $S_k$ is generated by $s_1, \ldots, s_{k-1}$ with relations

$$s_i^2 = 1, \quad s_is_{i+1}s_i = s_{i+1}s_is_{i+1}, \quad \text{and} \quad s_is_j = s_js_i \text{ for } j \neq i, i \pm 1.$$ 

4. In the group algebra of the symmetric group $\mathbb{C}S_k$ define

$$m_j = s_{1j} + s_{2j} + \cdots + s_{j-1,j},$$

where $s_{ij}$ is the transposition that switches $i$ and $j$. Let $m_1 = 0$.

   a. Show that $m_1 + \cdots + m_k$ is an element of the center of $\mathbb{C}S_k$.

   b. Show that $m_im_j = m_jm_i$ for all $1 \leq i, j \leq k$.

5. Construct explicitly some modules for $\mathbb{C}S_k$, which have a basis of eigenvectors for the $m_i$. Do this by describing, explicitly, the action of the $s_i$ and the $m_i$ on the basis vectors.

   a. Be sure to prove that the modules you construct are $S_k$-modules (by showing that the formulas for the action satisfy the necessary relations).

   b. Show that the modules you have constructed are irreducible.

   c. Show that the modules you constructed are pairwise nonisomorphic.

   d. Show that you have constructed all the irreducible $S_k$-modules.
6. Use the modules constructed in Problem 5 (or find an alternative method) to determine (with proof) the Bratelli diagram for the tower of algebras

\[ \mathcal{C}S_1 \subseteq \mathcal{C}S_2 \subseteq \mathcal{C}S_3 \subseteq \cdots \]