1. Algebras

(1) Define *algebra* and *Lie algebra*.

(2) Define *group algebra* and *enveloping algebra*.

(3) Define *basis* and *multiplication table* of an algebra.

(4) Define *presentation* of an algebra (by generators and relations).

(5) Write (with proof) a basis and the multiplication rule for the Temperley-Lieb algebras \( \text{TL}_1, \text{TL}_2, \text{TL}_3, \text{TL}_4 \). To do this, you will first have to define these algebras.

(6) Write (with proof) a basis and the multiplication rule for the matrix algebras \( M_1(\mathbb{C}), M_2(\mathbb{C}), M_3(\mathbb{C}), M_4(\mathbb{C}) \). To do this, you will first have to define these algebras.

(7) Write (with proof) a basis and the multiplication rule for the group algebras of the symmetric groups \( \mathbb{C}S_1, \mathbb{C}S_2, \mathbb{C}S_3, \mathbb{C}S_4 \). To do this, you will first have to define these algebras.

(8) Write (with proof) a basis and the multiplication rule for the Brauer algebras \( W_1, W_2, W_3, W_4 \). To do this, you will first have to define these algebras.

(9) Write (with proof) a basis and the multiplication rule for the Iwahori-Hecke algebras \( H_1, H_2, H_3, H_4 \) (of finite type A). To do this, you will first have to define these algebras.
Write (with proof) a basis and the multiplication rule for the tensor algebras. To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the symmetric algebras. To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the exterior algebras. To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the group algebras of the cyclic groups. To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the group algebras of the dihedral groups. To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the group algebras of the alternating groups. To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the group algebras of the tetrahedral group. To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the group algebras of the octahedral group. To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the group algebras of the icosahedral group. To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the group algebras of finite abelian groups. To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the group algebras of GL_1(ℂ), GL_2(ℂ), and GL_3(ℂ). To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the group algebras of GL_1(ℤ), GL_2(ℤ), and GL_3(ℤ). To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the group algebras of SL_1(ℂ), SL_2(ℂ), and SL_3(ℂ). To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the group algebras of SL_1(ℤ), SL_2(ℤ), and SL_3(ℤ). To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the enveloping algebras of g I_1(ℂ), g I_2(ℂ), and g I_3(ℂ). To do this, you will first have to define these algebras.

Write (with proof) a basis and the multiplication rule for the enveloping algebras of ℍ I_1(ℂ), ℍ I_2(ℂ), and ℍ I_3(ℂ). To do this, you will first have to define these algebras.

Provide and prove a presentation of the Temperley-Lieb algebras.

Provide and prove a presentation of the group algebras of the symmetric groups.

Provide and prove a presentation of the Brauer algebras.
Provide and prove a presentation of the Iwahori-Hecke algebras (of finite type A).

Provide and prove a presentation of the tensor algebras.

Provide and prove a presentation of the symmetric algebras.

Provide and prove a presentation of the exterior algebras.

Provide and prove a presentation of the Weyl algebras.

Provide and prove a presentation of the polynomial algebras.

Provide and prove a presentation of the algebra of continuous functions.

Provide and prove a presentation of the algebra of differentiable functions.

Provide and prove a presentation of the algebra of $C^r$ functions.

Provide and prove a presentation of the algebra of holomorphic functions.

Provide and prove a presentation of the algebra of meromorphic functions.

Provide and prove a presentation of the algebra of regular functions.

Provide and prove a presentation of the algebra of complex numbers.

Provide and prove a presentation of the algebra of quaternions.

Provide and prove a presentation of the algebra of the octonions.

Provide and prove a presentation of the Clifford algebras.

Provide and prove a presentation of the group algebras of the cyclic groups.

Provide and prove a presentation of the group algebras of the dihedral groups.

Provide and prove a presentation of the group algebras of the alternating groups.

Provide and prove a presentation of the group algebras of the tetrahedral group.

Provide and prove a presentation of the group algebras of the octahedral group.

Provide and prove a presentation of the group algebras of the icosahedral group.

Provide and prove a presentation of the group algebras of finite abelian groups.

Provide and prove a presentation of the group algebras of $GL_1(\mathbb{C})$, $GL_2(\mathbb{C})$, and $GL_3(\mathbb{C})$.

Provide and prove a presentation of the group algebras of $GL_1(\mathbb{Z})$, $GL_2(\mathbb{Z})$, and $GL_3(\mathbb{Z})$.

Provide and prove a presentation of the group algebras of $SL_1(\mathbb{C})$, $SL_2(\mathbb{C})$, and $SL_3(\mathbb{C})$.

Provide and prove a presentation of the group algebras of $SL_1(\mathbb{Z})$, $SL_2(\mathbb{Z})$, and $SL_3(\mathbb{Z})$.

Provide and prove a presentation of the enveloping algebras of $\mathfrak{g} I_1(\mathbb{C})$, $\mathfrak{g} I_2(\mathbb{C})$, and $\mathfrak{g} I_3(\mathbb{C})$.

Provide and prove a presentation of the enveloping algebras of $\mathfrak{s} I_1(\mathbb{C})$, $\mathfrak{s} I_2(\mathbb{C})$, and $\mathfrak{s} I_3(\mathbb{C})$. 
2. Representations

(1) Define representation.
(2) Define module.
(3) Define simple module.
(4) Define direct sum and indecomposable module.
(5) Show that the kernel of a homomorphism is a module.
(6) Show that the image of a homomorphism is a module.
(7) Show that submodules of the regular representation are left ideals.
(8) Classify and construct all left ideals of the Temperley-Lieb algebras $\text{TL}_1$, $\text{TL}_2$, $\text{TL}_3$, $\text{TL}_4$. Do the same for all the algebras in the previous section.

3. Categories

(1) Define category.
(2) Define functor and give some interesting examples of functors.
(3) Define abelian category, and linear category and 2-category.
(4) Define kernel and cokernel and image.
(5) Define exact sequence and short exact sequence and complex and acyclic complex.

4. Groups

(1) Define group.
(2) Define presentation of a group.

5. References

[Ra] A. Ram, Notes,