1. Classify and construct the finite dimensional simple modules for cyclic groups.

2. Classify and construct the finite dimensional simple modules for dihedral groups.

3. Classify and construct the finite dimensional simple modules for \( U_q \mathfrak{sl}_2 \), where \( U_q \mathfrak{sl}_2 \) is the algebra generated by \( E, F, K \pm 1 \), with relations
   
   \[ KEK^{-1} = q^2 E, \quad KFK^{-1} = q^{-2} F, \quad \text{and} \quad EF - FE = \frac{K - K^{-1}}{q - q^{-1}}. \]

4. Define the symmetric group (via permutations).

5. Show that \( S_k \) is generated by \( s_1, \ldots, s_{k-1} \) with relations
   
   \[ s_i^2 = 1, \quad s_is_{i+1}s_i = s_{i+1}s_is_{i+1}, \quad \text{and} \quad s_is_j = s_js_i \quad \text{for} \quad j \neq i, \ i \pm 1. \]

6. In the group algebra of the symmetric group \( \mathbb{C}S_k \) define
   
   \[ m_j = s_{1j} + s_{2j} + \cdots + s_{j-1,j}, \]
   
   where \( s_{ij} \) is the transposition that switches \( i \) and \( j \). Let \( m_1 = 0 \).

   a. Show that \( m_1 + \cdots + m_k \) is an element of the center of \( \mathbb{C}S_k \).

   b. Show that \( m_im_j = m_jm_i \) for all \( 1 \leq i, \ j \leq k \).