Representation Theory: Tuesday Branching rules

There are equivalences of categories:

\[ \{ \text{complex reductive} \} \mapsto \{ \text{compact} \} \mapsto \{ \mathbb{Z} \text{-reflection} \} \]

\[ \text{GL}_n(\mathbb{C}) \rightarrow \text{U}_n(\mathbb{C}) \rightarrow (\text{SU}, \mathbb{Z}^n) \]

\[ G \rightarrow K, \text{ the maximal compact subgroup} \rightarrow (W_0, \mathfrak{g}^*_2) \text{ with } W_0 = N(T)/T, \mathfrak{g}^*_2 = \text{Hom}(\mathfrak{t}, \mathbb{C}) \]

Hence there are equivalences:

\[ \{ \text{G-modules} \} \leftrightarrow \{ \mathfrak{g} \text{-modules} \} \leftrightarrow \{ (W_0, \mathfrak{g}^*_2) \text{ crystals} \} \]

\[ L(\lambda) \mapsto L(\lambda) \mapsto B(\lambda) \]

Suppose \( H \leq G \) (or \( L \leq K \)) (or \( (W_0, \mathfrak{g}^*_2) \leq (W_0, \mathfrak{g}^*_2) \))

Let \( L(\lambda) \) be a simple \( G \)-module:

Describe the decomposition of

\[ \text{Res}_H^G \left( L(\lambda) \right) \]

into irreducibles.

This should be equivalent to

\[ \text{Res}_{W_0}^{W_0} \left( L(\lambda) \right) \]

\[ (W_0, \mathfrak{g}^*_2) (B(\lambda)) \]
Example: The adjoint representation of $SU_5(C)$ restricted to $SU_3 \times SU_2 \times U_1$. 

Diagram showing the relationship between different representations of $SU_5(C)$.