$G = \text{Gl}_n(\mathbb{C})$ with $T = \left\{ \left( \begin{smallmatrix} a_1 & \cdots & a_n \end{smallmatrix} \right) \mid a_1, a_2, \ldots, a_n \in \mathbb{C}^n \right\}$

Then

$\zeta^v = \text{Hom}(\mathbb{C}^n, T) = \mathbb{Z} \text{span } \zeta^{v_1}, \zeta^{v_2}, \ldots, \zeta^{v_n}$

$\zeta^{v_i} : \mathbb{C}^n \to T \quad \leftarrow \quad e^v$

$z \mapsto (1, 1, \ldots)$

so that, if $\mu^v = \mu^{v_1} + \ldots + \mu^{v_n}$ then

$\zeta^{\mu^v}(z) = \zeta^{v_1}(z)^{\mu_1} \ldots \zeta^{v_n}(z)^{\mu_n}$

Then

$\zeta^* = \text{Hom}(T, \mathbb{C}^n) = \mathbb{Z} \text{span } \zeta^{e_1}, \zeta^{e_2}, \ldots, \zeta^{e_n}$

$X^i : T \to \mathbb{C}^n \quad \leftarrow \quad e^i$

$\left( \begin{smallmatrix} a_1 \\ \vdots \\ a_n \end{smallmatrix} \right) \mapsto a_i$

so that, if $X = X^1 e_1 + \ldots + X^n e_n$ then

$X X^i / a_i = a_1^{X^i} a_2^{X^i} \ldots a_n^{X^i}$

Composition gives a pairing

$\zeta^v \times \zeta^w = \text{Hom}(\mathbb{C}^n, T) \times \text{Hom}(T, \mathbb{C}^n) \to \text{Hom}(\mathbb{C}^n, \mathbb{C}^n) = \mathbb{C}$

$(h, X) \mapsto X \circ h : \mathbb{C}^n \to \mathbb{C}^n$

$(\zeta^{v_i}, \zeta^{w_j}) \mapsto z^{<v_i, w_j>}$

where $\langle \zeta^{v_i}, \zeta^{w_j} \rangle = \delta_{ij}$. 
The Weyl group is $W_0 = N_0(T)/Z_0(T)$.

$Z_0(T) = T = \{ (a_1, \ldots, a_n) \mid a_1, \ldots, a_n \in \mathbb{C}^\times \}$

$N_0(T) = \{ \text{real matrices with exactly one nonzero} \}
\{ \text{entry in each row and each column} \}
\{ \text{and nonzero entries on } \mathbb{C}^\times \}

= \{ w (a_1, \ldots, a_n) \mid w \in S_n, a_1, \ldots, a_n \in \mathbb{C}^\times \}.

So $W_0 = N_0(T)/T = \{ WT \mid w \in S_n \} = S_n$.

Then $W_0 \times S_n$ acts on $\mathbb{C}^n$ linearly on

$\xi \mapsto \mathbb{C}\text{span}(\xi, \ldots, \xi)$ by permuting $\xi, \ldots, \xi$

$\xi' \mapsto \mathbb{C}\text{span}(\xi', \ldots, \zeta)$ by permuting $\zeta, \ldots, \zeta'$.

So $W_0 \times S_n$ and $W_0 \times S_n$ for $i,j \leq 1, 2, \ldots, n$ and $w \in S_n$.

Complex one parameter subgroups of $G$ are group homomorphisms $\mathbb{C} \to G$. 
Let $E_{ij}$ be the nxn matrix with 1 in the $(i,j)$-entry and zero elsewhere. Define

$$x_{ij} : C \to \operatorname{Gl}_n(C) \quad \text{where} \quad x_{ij}(t) = 1 + tE_{ij} = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$$

Then $x_{ij}(t)x_{ij}(s) = x_{ij}(t+s)$ so that $x_{ij}$ is a group homomorphism.

Define

$$C \to C^* \to \operatorname{Gl}_n(C) \quad \text{so that} \quad h_x(e^t) = 1 + (e^t - 1)E_{ii}.$$ 

Then $h_x(e^t)h_x(e^s) = h_x(e^{t+s})$.

Note that

$$\left( \frac{d}{dt} x_{ij}(t) \right)_{t=0} = E_{ij} \quad \text{and} \quad \left( \frac{d}{dt} (h_x(e^t)) \right)_{t=0} = e^0 E_{ii} = E_{ii}.$$

The Lie algebra of $\operatorname{Gl}_n(C)$ has a complex Lie group is

$$\mathfrak{gl}_n(C) = \mathbb{C} \text{-span}\{ E_{ij} \mid 1 \leq i \neq j \leq n \} = \mathfrak{h}_n(\mathbb{C}).$$

The exponential map is

$$\mathfrak{gl}_n \to \operatorname{Gl}_n(C) \quad \text{where} \quad e^X = 1 + X + \frac{X^2}{2!} + \frac{X^3}{3!} + \cdots.$$
The Lie algebra $\mathfrak{g} = g_{2n}$ has Cartan subalgebra

$$\mathfrak{h} = C\text{span} \{ E_{ii} \} = C \otimes \mathfrak{h}_2$$

and the 1-dimensional representations of $\mathfrak{h}$ are

$$\mathfrak{h}_c^* = \text{Hom}(\mathfrak{h}, C) = C\text{span} \{ \delta_i \} = C \otimes \mathfrak{h}_2^*$$

where

$$\delta_i : \mathfrak{h} \to C$$

$$\begin{pmatrix} a_i \neq 0 \\ 0 \end{pmatrix} \mapsto a_i \quad \text{for } i = 1, 2, \ldots, n.$$ 

If $M$ is a $\mathfrak{g}$-module then

$$M = \bigoplus_{\mu \in \mathfrak{h}_c^*} M_{\mu} \quad \text{where } M_{\mu} = \{ m \in M \mid x \in \mathfrak{g} \Rightarrow xM = \mu(x) m \}$$

The $\mu \in \mathfrak{h}_c^*$ such that $M_{\mu} \neq \{0\}$ are the weights of $M$. 