Title: Uniform-in-time stability result for doubly degenerate parabolic equations.

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Abstract: We consider a class of doubly degenerate parabolic equations
\[ \partial_t (\beta(u)) - \operatorname{div}(a(x, \nu(u), \nabla \zeta(u))) = f \] (1)
with homogeneous Dirichlet boundary conditions. The functions \( \beta \) and \( \zeta \) are Lipschitz-continuous and non-decreasing, but may contain plateaux. The operator \( -\operatorname{div}(a(x, s, \cdot)) \) is a Leray–Lions operator acting on \( W^{1,p}_0(\Omega) \), and \( \nu \) is defined by \( \nu' = \beta' \zeta' \). Particular cases of (1) are:

- Richards’ model of groundwater flow: \( \partial_t \beta(u) - \operatorname{div}(K(x, \beta(u))\nabla u) = f \). Here, \( u \) is the pressure and \( \nu(u) = \beta(u) \) is the saturation.
- Stefan’s model of melting material: \( \partial_t u - \operatorname{div}(K(x, \zeta(u))\nabla \zeta(u)) = f \). Here, \( u \) is the internal energy and \( \nu(u) = \zeta(u) \) is the temperature.
- Leray–Lions equations, with prototype the \( p \)-Laplace equation: \( \partial_t u - \operatorname{div}(|\nabla u|^{p-2} \nabla u) = f \). Such models are involved in non-Newtonian filtration. Here, \( \nu(u) = u \).

We will present a novel stability result for the solutions of (1) under perturbations of the data \( \beta, \zeta, a, f \), that is: if \( (\beta_\epsilon, \zeta_\epsilon, a_\epsilon, f_\epsilon) \) converge as \( \epsilon \to 0 \) to \( (\beta, \zeta, a, f) \) in a natural sense, and if \( u_\epsilon \) is a solution to (1) with \( (\beta_\epsilon, \zeta_\epsilon, a_\epsilon, f_\epsilon) \), then \( u_\epsilon \to u \) solution to (1). As usual, this stability result also gives the existence of solutions to the equation.

As demonstrated by the examples above, the value of \( \nu(u) \) at a given time \( T \) is the quantity of interest in practical applications. The main novelty of our result is to establish the uniform-in-time and strong-in-space convergence (i.e. a \( C([0, T]; L^2(\Omega)) \) convergence) of \( \nu_\epsilon(u_\epsilon) \), without assuming non-physical regularity properties on the solution – which allows in particular for discontinuous permeabilities and conductivities \( K(\cdot, s) \), as encountered in applications.

The techniques used to establish the stability result are:

1. Energy equalities (not mere estimates),
2. Aubin–Simon and Ascoli–Arzela (in weak topology) compactness theorems,
3. Compensated compactness,
4. Convexity properties,
5. Characterisation of uniform convergence.

Time allowing, the presentation will be concluded with a twist.