

Math 127 A 2

Winter quarter, 2005

Mid term exam 1

Question 1

Suppose that A and B are cuts of rational numbers. Explain why the definition $A + B = \{r + s; r \in A, s \in B\}$ gives a cut of rational numbers.

Question 2

Suppose that A and B are cuts of rational numbers. Explain why the definition $A - B = \{r - s; r \in A, s \in B\}$ does not give a cut of rational numbers.

Question 3

Prove that $\sqrt{6}$ is not a rational number. Use this to prove that $\sqrt{2} + \sqrt{3}$ is not a rational number.

Question 4

Which of the following are Cauchy sequences? Give a brief justification for your answers.

- $\{1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, \dots\}$
- $\{2, 2\frac{1}{4}, 2\frac{1}{9}, 2\frac{1}{16}, \dots\}$
- $\{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \dots\}$
- $\{1, -1\frac{1}{2}, +1\frac{1}{3}, -1\frac{1}{4}, \dots\}$

Question 5

Which of the following are inner products on the plane \mathbb{R}^2 ? Give a brief reason for your answers.

- $\langle (a, b), (x, y) \rangle = ax - by$
- $\langle (a, b), (x, y) \rangle = 2ax + 3by$

Question 6

Use question 5 to prove the inequality $(2ax + 3by)^2 \leq (2a^2 + 3b^2)(2x^2 + 3y^2)$