Math 127 A 2
Winter quarter, 2005
Mid term exam 1

Question 1
Suppose that $A$ and $B$ are cuts of rationals numbers. Explain why the definition $A + B = \{ r + s; r \in A, s \in B \}$ gives a cut of rational numbers.

Question 2
Suppose that $A$ and $B$ are cuts of rationals numbers. Explain why the definition $A - B = \{ r - s; r \in A, s \in B \}$ does not give a cut of rational numbers.

Question 3
Prove that $\sqrt{6}$ is not a rational number. Use this to prove that $\sqrt{2} + \sqrt{3}$ is not a rational number.

Question 4
Which of the following are Cauchy sequences? Give a brief justification for your answers.

- $\{1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, \ldots \}$
- $\{2, 2\frac{1}{4}, 2\frac{1}{9}, 2\frac{1}{16}, \ldots \}$
- $\{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots \}$
- $\{1, -1\frac{1}{2}, +1\frac{1}{3}, -1\frac{1}{4}, \ldots \}$

Question 5
Which of the following are inner products on the plane $\mathbb{R}^2$? Give a brief reason for your answers.

- $\langle (a, b), (x, y) \rangle = ax - by$
- $\langle (a, b), (x, y) \rangle = 2ax + 3by$

Question 6
Use question 5 to prove the inequality $(2ax + 3by)^2 \leq (2a^2 + 3b^2)(2x^2 + 3y^2)$