Math 127 A 2

Winter quarter, 2005

Practice questions

1 Is the following a cut - if not explain how to construct a cut representing $\sqrt{2} - \sqrt{3}$?

Let $A = \{q \in \mathbb{Q} : q^2 < 2\}$, $B = \{q' \in \mathbb{Q} : q'^2 < 3\}$, $C = \{q - q' : q \in A, q' \in B\}$

2 Define a norm on a vector space. Which of the following are norms, where $v = (x, y) \in \mathbb{R}^2$?

- $|v| = max\{|x|, |y|\}$
- $|v| = x^2 + y^2$

3 Explain why the collection of finite sequences of integers is countable.

Is the collection of decimal expansions of numbers in $[0, 1]$ only involving integers 1 and 2 countable or uncountable? So a number of the form $0.12211112\ldots$ is in the collection but the number 0.231 is not in the collection. Explain.

4 Explain why the set $\{(x, y) : x^2 + 3y^2 = 1$ is a compact subset of $\mathbb{R}^2$. Find a subset of $\mathbb{R}^2$ for which this set is the boundary. Can a single point be the boundary of a set in $\mathbb{R}^2$?

5 Explain why the graph of a continuous function $f : \mathbb{R} \to \mathbb{R}$ is a connected subset of the plane. Is the converse true, i.e if the graph is connected, does this imply the function is continuous?

6 Define completeness for a metric space. Which of the following examples is complete?

- The unit circle in the plane i.e $\{(x, y) : x^2 + y^2 = 1\}$.
- Any compact subset of a metric space.
- Any compact subset of a complete metric space.
- All the irrational numbers, i.e $\mathbb{R} \setminus \mathbb{Q}$.