

**620-142 Mathematics B**  
**Assignment 1**  
**Due: 2pm, Friday, April 4**

Please leave your assignment in your tutor's box located near the north entrance to the Richard Berry building. Make sure that you have written your name, your student number, your tutor's name, and your tutorial time on the front page.

With this first assignment, you need to include a signed plagiarism form.

1. (a) If  $n$  is an even integer, show that  $n^3$  is divisible by 8.  
(b) If  $a$  is a factor of  $b$ , and  $b$  is a factor of  $c$ , show that  $a$  is a factor of  $5b - 3c$ .
2. (a) (i) Construct the multiplication table for  $\mathbb{Z}_9$ .  
(ii) Use your table in (i) to solve the equation

$$7x + 4 \equiv 1 \pmod{9}.$$

- (iii) Use your table in (i) to determine which elements of  $\mathbb{Z}_9$  have no multiplicative inverse, and to write down the multiplicative inverses of the other elements.
- (b) Calculate the order of 2 in  $\mathbb{Z}_9$ .
3. (a) Use the Euclidean Algorithm to find the greatest common divisor of 44 and 13.  
(b) Use your working in (a) to assist you in finding integers  $x$  and  $y$  such that

$$44x + 13y = 1.$$

- (c) Use your answer from (b) to write down the value of  $13^{-1}$  in  $\mathbb{Z}_{44}$ .
4. (a) Calculate  $\phi(77)$ .  
(b) Use Fermat's Little Theorem to assist you in calculating  $5^{36}$  in  $\mathbb{Z}_{13}$ .  
(c) Use binary powering to compute  $5^{43}$  in  $\mathbb{Z}_{49}$ .
5. Use mathematical induction to prove that

$$1 + 2 \times 2^1 + 3 \times 2^2 + 4 \times 2^3 + \dots + n \times 2^{n-1} = 1 + (n - 1)2^n$$

for all positive integers  $n$ .

**Challenge problem (Not for assessment)**

(Chocolate bars will be given for the best solutions!)

Prove that there is no non-constant polynomial  $P(x)$  with integer coefficients such that  $P(n)$  is prime for all integers  $n$ .