

620-142 Mathematics B
Assignment 3
Due: 2pm, Friday, May 2

Please leave your assignment in your tutor's box located near the north entrance to the Richard Berry building. Make sure that you have written your name, your student number, your tutor's name, and your tutorial time on the front page.

You should give **complete explanations** for all your answers.

1. Let

$$A = \begin{bmatrix} 1 & 1 & 2 & -1 & 1 \\ 2 & 2 & 4 & -2 & 2 \\ 3 & 3 & 6 & 5 & 11 \end{bmatrix}.$$

- (a) Determine the rank of A .
 - (b) Find a basis for, and the dimension of, the column space of A .
 - (c) Find a basis for, and the dimension of, the row space of A .
 - (d) Find a basis for, and the dimension of, the solution space of A .
2. In each part of this question, determine whether H is a subspace of the vector space V . (For each part, give a complete proof using the subspace theorem, or else give a specific example to show that some subspace property fails.)
- (a) $V = M^{2,2}$, $H = \{A \in M^{2,2} : \det A = 1\}$
 - (b) $V = \mathcal{P}_2$, $H = \{p \in \mathcal{P}_2 : p(3) = 0\}$

3. (a) Consider the 2×2 matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 & 4 \\ 2 & 0 \end{bmatrix}, C = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Determine whether these matrices are linearly dependent or linearly independent.

- (b) Determine whether the set of polynomials $\{1 - x + x^2, 3 - x^2, 1 + 2x - 3x^2\}$ spans \mathcal{P}_2 .
 - (c) If p_1, p_2, \dots, p_k are linearly independent polynomials in \mathcal{P}_n , write down a mathematical relationship between k and n .
4. (a) The management of a chain of retail stores wishes to introduce barcodes for each item stocked by the stores. It is proposed to introduce an 11 digit code for each item, with the first 10 digits $x_1, x_2, x_3, \dots, x_{10}$ being the information digits and the eleventh digit x_{11} being the check digit. Each digit is to be a decimal digit, that is, one of $0, 1, 2, \dots, 8, 9$. The check digit is calculated so that

$$x_1 - x_2 + x_3 - x_4 + x_5 - x_6 + x_7 - x_8 + x_9 - x_{10} + x_{11} \equiv 0 \pmod{10}.$$

- (i) Determine the check digit if the information digits are 0369147258.
- (ii) Explain why the code is not able to detect the interchange of two adjacent digits for certain values of the digits.

Please turn over.

(b) (i) Find a basis for the solution space of

$$\begin{aligned}x_1 + x_4 + x_5 &= 0 \\x_2 + x_5 + x_6 &= 0 \\x_3 + x_4 + x_5 + x_6 &= 0\end{aligned}$$

using \mathbb{Z}_2 arithmetic.

- (ii) Write down all the vectors in the solution space of (i).
(iii) Consider the code with check matrix

$$\begin{bmatrix}1 & 0 & 0 & 1 & 1 & 0 \\0 & 1 & 0 & 0 & 1 & 1 \\0 & 0 & 1 & 1 & 1 & 1\end{bmatrix}.$$

Use this matrix to correct any transmission error in the received message 111110.
What can you conclude if the message 111111 is received?

Challenge problem (Not for assessment)

(Edible prizes will be awarded for the best solutions!)

An *algebraic curve* in \mathbb{R}^2 is the set of zeros of a non-zero polynomial $f(x, y)$ in two variables

$$\{(x, y) \in \mathbb{R}^2 : f(x, y) = 0\}.$$

(For example, the unit circle is an algebraic curve, consisting of the points such that $x^2 + y^2 - 1 = 0$.)

Consider a *polynomial path* in \mathbb{R}^2 , that is, a parametrized curve $x = x(t)$, $y = y(t)$, where x and y are polynomials in t . (For example: $x = t$, $y = t^2$ is a polynomial path which describes a parabola.)

- (a) Show that the polynomial functions $x(t)^i y(t)^j$, $0 \leq i, j \leq n$ are linearly dependent when n is sufficiently large. [Note: the value of n you need to choose will depend on the degrees of $x(t)$ and $y(t)$.]
(b) Deduce that every polynomial path lies on a real algebraic curve.
(c) Find an algebraic curve which contains the polynomial path: $x = t^2 + t$, $y = t^3$.