Answers: Inner products

1. For all vectors $\mathbf{x}, \mathbf{y}, \mathbf{z}$ and all scalars α:
   (a) $\langle \mathbf{x}, \mathbf{y} \rangle = \langle \mathbf{y}, \mathbf{x} \rangle$
   (b) $\langle \mathbf{x} + \mathbf{y}, \mathbf{z} \rangle = \langle \mathbf{x}, \mathbf{z} \rangle + \langle \mathbf{y}, \mathbf{z} \rangle$
   (c) $\langle \alpha\mathbf{x}, \mathbf{y} \rangle = \alpha\langle \mathbf{x}, \mathbf{y} \rangle$
   (d) $\langle \mathbf{x}, \mathbf{x} \rangle \geq 0$ for all $\mathbf{x}$ and $\langle \mathbf{x}, \mathbf{x} \rangle = 0 \Rightarrow \mathbf{x} = \mathbf{0}$.

2. $\mathbf{x} - \mathbf{y} = (-1, -1, -1)$ so

$$\langle \mathbf{x} - \mathbf{y}, \mathbf{x} - \mathbf{y} \rangle = (-1) \times (-1) + (-1) \times (-1) + 2 \times (-1) \times (-1) = 4$$

   and

$$\|\mathbf{x} - \mathbf{y}\| = \sqrt{\langle \mathbf{x} - \mathbf{y}, \mathbf{x} - \mathbf{y} \rangle} = 2.$$

3. $\langle \mathbf{x}, \mathbf{y} \rangle = 2, \quad \|\mathbf{x}\| = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle} = \sqrt{3}$, and $\|\mathbf{y}\| = \sqrt{\langle \mathbf{y}, \mathbf{y} \rangle} = \sqrt{5}$ so

$$\cos\theta = \frac{\langle \mathbf{x}, \mathbf{y} \rangle}{\|\mathbf{x}\|\|\mathbf{y}\|} = \frac{2}{\sqrt{15}}.$$

4. 0

5. One possibility is

$$\langle a_0 + a_1x + a_2x^2, b_0 + b_1x + b_2x^2 \rangle = a_0b_0 + a_1b_1 + a_2b_2.$$

6. One possibility is

$$\left\langle \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}, \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \right\rangle = a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4.$$