## Answers: linear combination questions

1. Sketch the vectors $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

2. The vectors are linearly dependent since one is a multiple of the other. Or, since

$$1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

3. The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ are linearly independent if

$$\alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \ldots + \alpha_n\mathbf{v}_n = \mathbf{0} \text{ implies } \alpha_i = 0 \text{ for } i = 1, \ldots, n.$$

   Alternatively the matrix equation, $A\mathbf{x} = \mathbf{0}$ has $\mathbf{x} = \mathbf{0}$ as the only solution where

$$A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_n \end{bmatrix}$$

   is the matrix with columns $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n$ and

$$\mathbf{x} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix}.$$

4. We have 3 vectors in $\mathbb{R}^2$. Since number of vectors $> \dim(\mathbb{R}^2) = 2$ the vectors must be linearly dependent.

5. We have 3 vectors in the vector space $\mathcal{P}_1$ of all polynomials of degree $\leq 1$. Since number of vectors $> \dim(\mathcal{P}_2) = 2$ the vectors must be linearly dependent.