Answers: linear combination questions

1. Sketch the vectors \[ \begin{bmatrix} 4 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 \\ 2 \end{bmatrix} \].

2. The vectors are linearly dependent since one is a multiple of the other. Or, since

\[ \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \]

3. The vectors \( \{v_1, v_2, \cdots, v_n\} \) are linearly independent if

\[ \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = \mathbf{0} \] implies \( \alpha_i = 0 \) for \( i = 1, \ldots, n. \)

Alternatively the matrix equation, \( Ax = \mathbf{0} \) has \( x = \mathbf{0} \) as the only solution where

\[ A = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \]

is the matrix with columns \( v_1, v_2, \cdots, v_n \) and

\[ x = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix}. \]

4. We have 3 vectors in \( \mathbb{R}^2 \). Since number of vectors > dim(\( \mathbb{R}^2 \)) = 2 the vectors must be linearly dependent.

5. We have 3 vectors in the vector space \( P_1 \) of all polynomials of degree \( \leq 1. \) Since number of vectors > dim(\( P_2 \)) = 2 the vectors must be linearly dependent.