Answers: Matrix equations, row space, column space

1. $\begin{bmatrix} 2 & 3 & 0 \\ 1 & -1 & 1 \end{bmatrix}$

2. $\text{rank}A = 2$. $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.
(The number of free parameters $= 3 - 2 = 1 > 0$.

3. The vectors are linearly dependent if $A\mathbf{x} = \mathbf{0}$ has a non-zero solution. This holds by (2) above.

4. The column space of A consists of all linear combinations of the columns of A.
The row space of A consists of all linear combinations of the rows of A.

5. The column space of A has a basis consisting of the columns of A corresponding to leading entries (pivots) in the row echelon form:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} \right\}.$$

The row space of A has a basis consisting of the non-zero rows in the row echelon form:

$$\left\{ \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \right\}.$$

For the matrix A,
column rank $= \dim(\text{column space}) = 2$ and row rank $= \dim(\text{row space}) = 2$.
Hence

$$\text{column rank} = \text{row rank} = \text{rank}A = 2,$$

as expected.