# Solution to Revision Number 1

**Q4. Solution**

(i) We know that

dim (column space of $A$) = rank($A$).

From the matrix $B$, which is the reduced row echelon form of $A$, rank($A$) = 2. Hence dim (column space of $A$) = 2.

(ii) The vectors

$$(1, -1, 2), \quad (1, 1, -3), \quad (4, -2, 3)$$

are columns of the matrix $A$. Since the dimension of the column space of $A$ is 2, these vectors do not span $\mathbb{R}^3$ (which has dimension 3).

(iii) The vectors

$$(1, -1, 2), \quad (1, 1, -3), \quad (3, -1, 1)$$

are columns of the matrix $A$. Since the dimension of the column space of $A$ is 2, these vectors cannot be linearly independent, for if they were the dimension of the column space would be 3.

(iv) We want to find constants $a$ and $b$ such that

$$a \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}.$$

In augmented matrix form this reads

$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ -1 & 1 & -2 \\ 2 & -3 & 3 \end{array}\right].$$

Notice that this matrix consists of columns $1, 3, 4$ of $A$. So its row echelon form consists of columns 1,3,4 of $B$:

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right].$$

This gives $b = 1$ and $a = 3$.