REVIEW QUIZ

1. **[9 marks]** After performing several row operations, you can find that the following two matrices are row-equivalent:

$$A = \begin{bmatrix} 1 & 2 & -4 \\ 1 & 1 & -1 \\ 2 & 0 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}.$$

(There is no need for you to perform these row operations.) Using this information, answer the following:

   (a) Do the vectors $(1, 1, 2), (2, 1, 0), (-4, -1, 4)$ form a basis for $\mathbb{R}^3$?

   (b) What is the rank of A?

   (c) Can you write $(-4, -1, 4)$ as a linear combination of $(1, 1, 2)$ and $(2, 1, 0)$? If so, do it.

   (d) What is the nullity of A?

   (e) Find a basis for the null space of A.

   (f) Are the vectors $(1, 2, -4), (1, 1, -1), (2, 0, 4)$ linearly independent?

   (g) A is the standard matrix of a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$. Find $T(1, 2, 3)$ and a basis for the image of T.

2. **[11 marks]** Consider the symmetric matrix

$$C = \begin{bmatrix} 5 & -4 & 2 \\ -4 & 5 & 2 \\ 2 & 2 & 8 \end{bmatrix}.$$

Its characteristic polynomial is $\det(C - \lambda I) = -81\lambda + 18\lambda^2 - \lambda^3$. (There is no need to check this statement.)

   (a) Fred says that the vector $(2, 0, 1)$ is an eigenvector of C. Is he correct?

   (b) What are the eigenvalues of C?

   (c) Find bases for the eigenspaces of C. The fact that

$$C \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

   may save you time.

   (d) Find an orthogonal matrix Q which diagonalizes C.