1. **[9 marks]** After performing several row operations, you can find that the following two matrices are row-equivalent:

\[
A = \begin{bmatrix}
1 & 2 & -4 \\
1 & 1 & -1 \\
2 & 0 & 4
\end{bmatrix}, \quad B = \begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & -3 \\
0 & 0 & 0
\end{bmatrix}.
\]

(There is no need for you to perform these row operations.) Using this information, answer the following:

(a) Do the vectors \((1, 1, 2), (2, 1, 0), (-4, -1, 4)\) form a basis for \(\mathbb{R}^3\)?

(b) What is the rank of \(A\)?

(c) Can you write \((-4, -1, 4)\) as a linear combination of \((1, 1, 2)\) and \((2, 1, 0)\)? If so, do it.

(d) What is the nullity of \(A\)?

(e) Find a basis for the null space of \(A\).

(f) Are the vectors \((1, 2, -4), (1, 1, -1), (2, 0, 4)\) linearly independent?

(g) \(A\) is the standard matrix of a linear transformation \(T : \mathbb{R}^3 \to \mathbb{R}^3\). Find \(T(1, 2, 3)\) and a basis for the image of \(T\).

2. **[11 marks]** Consider the symmetric matrix

\[
C = \begin{bmatrix}
5 & -4 & 2 \\
-4 & 5 & 2 \\
2 & 2 & 8
\end{bmatrix}.
\]

Its characteristic polynomial is \(\det(C - \lambda I) = -81\lambda + 18\lambda^2 - \lambda^3\). (There is no need to check this statement.)

(a) Fred says that the vector \((2, 0, 1)\) is an eigenvector of \(C\). Is he correct?

(b) What are the eigenvalues of \(C\)?

(c) Find bases for the eigenspaces of \(C\). The fact that

\[
C \begin{bmatrix}
-2 \\
-2 \\
1
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

may save you time.

(d) Find an orthogonal matrix \(Q\) which diagonalizes \(C\).