QUIZ SOLUTIONS

1 (a) NO (1)

(b) 2 (1)

(c) $(-4,-1,4) = 2(1,1,2) - 3(2,1,0)$ (1)

(d) 1 (1)

(e) $x_3 = \lambda$

$x_1 = -2\lambda$

$x_2 = 3\lambda$

$x_3 = \lambda$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \lambda \underbrace{\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}}_{\uparrow\ \text{basis}} \quad (2)$$

(f) These vectors are in the row (1) space, which has (from B) dim. 2. $\therefore$ NOT independent.

$A\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(g) $T(1,2,3) = (-7,0,14)$ (1)

Basis = 1st 2 cols of A: $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$ (1)

2 (a) $C\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \\ \cdots \end{bmatrix}$ No! (1)

(b) $\det(C - \lambda I) = -\lambda(\lambda - 9)^2$

$\therefore \lambda = 0, \; \lambda = 9$ (mult'y 2) (2)

(c) $\underline{\lambda = 9}$: $\begin{bmatrix} -4 & -4 & 2 \\ -4 & -4 & 2 \\ 2 & 2 & -1 \end{bmatrix} \sim \begin{bmatrix} 2 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{array}{ccc} \mu_1 & \mu_2 & \\ \end{array}\begin{bmatrix} 1 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$-1 \quad \frac{1}{2}$ $\quad x_1 = -\mu_1 + \frac{1}{2}\mu_2$

$\cancel{0} \quad 0$ $\quad x_2 = \mu_1$

$0, \quad 1$ $\quad x_3 = \mu_2$

$\therefore \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$ (3)

$\underline{\lambda = 0}$ $\left\{ \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix} \right\}$ (1)

(d) Apply G-S to first basis: (2)

$\underset{\sim}{u}_1 = \frac{1}{\sqrt{2}}(-1,1,0)$ (1) $\underset{\sim}{u}_2 = \frac{\sqrt{2}}{3}\left(\frac{1}{2}, \frac{1}{2}, 2\right)$

Normalise vector for $\lambda = 0$:

$\frac{1}{3}(-2,-2,1)$ (1)

$$\therefore Q = \begin{bmatrix} -2/3 & -1/\sqrt{2} & \sqrt{2}/6 \\ -2/3 & 1/\sqrt{2} & \sqrt{2}/6 \\ 1/3 & 0 & \sqrt{8}/3 \end{bmatrix}$$

$\uparrow$ $\lambda = 0$ $\qquad \underbrace{\qquad\qquad}_{\lambda = 9.}$