

## Answers: Span, basis and dimension

1.  $\langle \mathbf{v}_1, \mathbf{v}_2 \rangle = \text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$  is the set of all linear combinations of  $\mathbf{v}_1$  and  $\mathbf{v}_2$ :

$$\{\alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 : \alpha_1, \alpha_2 \in \mathbb{R}\}.$$

2.  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  is a basis.

3. dimension = number of basis vectors = 3.

4. The span of the 3 given vectors is a subspace of  $\mathbb{R}^3$ , so the maximum possible dimension is  $\dim \mathbb{R}^3 = 3$ .

5. A basis is

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \right\}.$$

6.  $\langle \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \rangle = \mathbb{R}^3$ . (Any 3 linearly independent vectors in the 3-dimensional vector space  $\mathbb{R}^3$  form a basis, hence span  $\mathbb{R}^3$ .)