620-151 Introduction to Biomedical Mathematics
Derivatives

1. Derivatives and slopes: see diagram in lecture notes.

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \lim_{h \to \infty} \frac{f(x+h) - f(x)}{h}$$

   See BZB 6thEd p555, 7thEd p546, 8thEd p551.

2. Derivatives from first principles: see example in lecture notes and some in the text book. In practive we seldom find derivatives from first principles using the limit formula above. It is easier and quicker to have a short list of rules. The following rules can be constructed from the above formula but it is not trivial.

3. Short list of derivative rules:

$y = f(x)$	$y' = f'(x)$	Type	BZB refs		
			6th Ed	7th Ed	8th Ed
x^k	kx^{k-1}	polynomial	569	559	562
e^{kx}	ke^{kx}	exponential	725	703	691
$\log_e(kx)$	$1/x$	logarithm (base e)	725	703	691
$\sin(kx)$	$k\cos(kx)$	sine	$\circ$	$\circ$	$\circ$
$\cos(kx)$	$-k\sin(kx)$	cosine	$\circ$	$\circ$	$\circ$
$u + v$	$u' + v'$	sum	572	562	565
uv	$u'v + uv'$	product	582	571	574
u/v	$(u'v - uv')/v^2$	quotient	584	573	576
u^k	$ku^{k-1} \cdot u'$	power	591	579	583
$f(g(x))$	$f'(g(x)) \cdot g'(x)$	chain	741	718	705

   Complete statements of these rules are in BZB. *eg* the sum rule:

   - if $f(x) = u(x) + v(x)$ then $f'(x) = u'(x) + v'(x)$.
   - Alternatively — if $y = u(x) + v(x)$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}u}{\mathrm{d}x} + \frac{\mathrm{d}v}{\mathrm{d}x}$.

4. Chain Rule (alternative form):

   - Let $y = f(u)$ and $u = g(x)$. Define the composite function $y = f(g(x))$. Then the derivative is

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x} \qquad \text{or} \qquad (f(g(x)))' = f'(g(x)) \cdot g'(x)$$