4. Use the simplex method to solve the following standard maximum problem.

$$\text{Maximise} \qquad g = 12y_2 + 13y_3$$

$$\begin{array}{lrcrcrl} \text{subject to} & 2y_1 & + & 2y_2 & & & \leq 6 \\ & 12y_1 & + & 6y_2 & + & 12y_3 & \leq 24 \\ & 15y_1 & & & - & 16y_3 & \leq 8 \end{array}$$

with $y_1 \geq 0$, $y_2 \geq 0$ and $y_3 \geq 0$.

At each step show clearly the row operation(s) that you perform and clearly circle the pivot element. Inspect your final tableau and state the maximum possible value of g and all the values of (y_1, y_2, y_3) for which this maximum occurs.

We introduce slack variables s_1, s_2, s_3 so that the problem constraints become

$$2y_1 + 2y_2 + s_1 = 6,$$

$$12y_1 + 6y_2 + 12y_3 + s_2 = 24,$$

and $$15y_1 - 16y_3 + s_3 = 8,$$

where $s_1 \geq 0$, $s_2 \geq 0$, and $s_3 \geq 0$.

We rewrite the system as a simplex Tableaux

y_1	y_2	y_3	s_1	s_2	s_3	P	RHS	Quotients
2	2	0	1	0	0	0	6	—
12	6	12	0	1	0	0	24	$2 = \frac{24}{12}$
15	0	−16	0	0	1	0	8	—
0	−12	−13	0	0	0	1	0	

pivot column

$\frac{1}{12}R_2 \to R_2$

Pivot element is circled.

$$\sim\left[\begin{array}{ccccccc|c} 2 & 2 & 0 & 1 & 0 & 0 & 0 & 6 \\ 1 & \frac{1}{2} & 1 & 0 & \frac{1}{12} & 0 & 0 & 2 \\ 15 & 0 & -16 & 0 & 0 & 1 & 0 & 8 \\ \hline 0 & -12 & -13 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ 16R_2+R_3 \to R_3 \\ 12R_2+R_4 \to R_4 \end{array}$$

$$\sim\left[\begin{array}{ccccccc|c} 2 & 2 & 0 & 1 & 0 & 0 & 0 & 6 \\ 1 & \frac{1}{2} & 1 & 0 & \frac{1}{12} & 0 & 0 & 2 \\ 31 & 8 & 0 & 0 & \frac{4}{3} & 1 & 0 & 40 \\ \hline 13 & -\frac{11}{2} & 0 & 0 & \frac{13}{12} & 0 & 1 & 26 \end{array}\right]$$

Quotients

$\frac{6}{2} = 3$

$2/\frac{1}{2} = 4$

$40/8 = 5$

$\frac{1}{2}R_1 \to R_1$

↑ pivot column

$$\sim\left[\begin{array}{ccccccc|c} 1 & 1 & 0 & \frac{1}{2} & 0 & 0 & 0 & 3 \\ 1 & \frac{1}{2} & 1 & 0 & \frac{1}{12} & 0 & 0 & 2 \\ 31 & 8 & 0 & 0 & \frac{4}{3} & 1 & 0 & 40 \\ \hline 13 & -\frac{11}{2} & 0 & 0 & \frac{13}{12} & 0 & 1 & 26 \end{array}\right] \begin{array}{l} \\ -\frac{1}{2}R_1+R_2 \to R_2 \\ -8R_1+R_3 \to R_3 \\ \frac{11}{2}R_1+R_4 \to R_4 \end{array}$$

$$\sim \left[\begin{array}{ccccccc|c} 1 & 1 & 0 & \frac{1}{2} & 0 & 0 & 0 & 3 \\ \frac{1}{2} & 0 & 1 & -\frac{1}{4} & \frac{1}{12} & 0 & 0 & \frac{1}{2} \\ 23 & 0 & 0 & -4 & \frac{4}{3} & 1 & 0 & 16 \\ \hline \frac{37}{2} & 0 & 0 & \frac{11}{4} & \frac{13}{12} & 0 & 1 & \frac{85}{2} \end{array}\right]$$

STOP since there are no negative elements in bottom row.

We deduce that the maximum value of g is $\frac{85}{2}$ occurring at $(y_1, y_2, y_3) = (0, 3, \frac{1}{2})$.

The slack variables at this point have values $(s_1, s_2, s_3) = (0, 0, 16)$.

④

Answer check: (We note that $y_1, y_2, y_3 \geq 0$ at $(y_1, y_2, y_3) = (0, 3, \frac{1}{2})$).

If we label

$$2y_1 + 2y_2 \leq 6 \qquad (i)$$

$$12y_1 + 6y_2 + 12y_3 \leq 24 \qquad (ii)$$

$$15y_1 - 16y_3 \leq 8 \qquad (iii)$$

We note that at $(y_1, y_2, y_3) = (0, 3, \frac{1}{2})$

LHS of (i) $= 2 \times 0 + 2 \times 3 = 6 \leq 6 =$ RHS of (i)

(and $S_1 = 0$) so OK ✓

LHS of (ii) $= 12 \times 0 + 6 \times 3 + 12 \times \frac{1}{2}$

$= 18 + 6 = 24 \leq 24 =$ RHS of (ii) ✓

(and $S_2 = 0$) so OK

LHS of (iii) $= 15 \times 0 - 16 \times \frac{1}{2} = -8 \leq 8 =$ RHS of (iii) ✓

So OK.

(and $S_3 = 8 - (-8) = 16$).

Now $g = 12y_2 + 13y_3 = 12 \times 3 + 13 \times \frac{1}{2}$

$= 36 + \frac{13}{2} = \frac{72 + 13}{2} = \frac{85}{2}$

So OK. ✓