620-151 — linear programming by graphical methods.
example problem.

The problem:

- A manufacturer makes shirts and jackets.
- They have just 1 press and 1 sewing machine.
- Each shirt needs 4 minutes sewing and 1 minute pressing.
- Each jacket needs 2 minutes sewing and 3 minutes pressing.
- The sewing machine can only operate for 33 hours 20 minutes = 2000 minutes per week.
- The press can only operate for 25 hours = 1500 minutes per week.
- The profit made is \$5 per shirt and \$6 per jacket.
- How many shirts and how many jackets should be made per week to maximise profit?

The mathematical model:

- Let x be the number of shirts made in a week.
- Let y be the number of jackets made in a week.
- Cannot make a negative number of shirts or jackets, so

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \end{aligned}$$

- Total sewing time is less than 2000 minutes per week, so

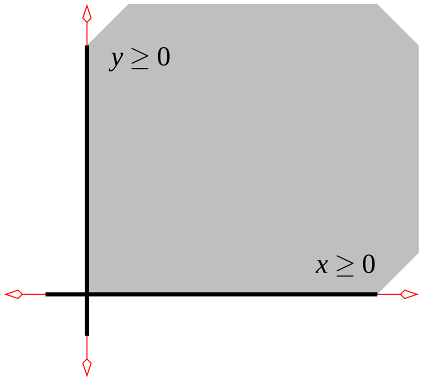
$$4x + 2y \leq 2000$$

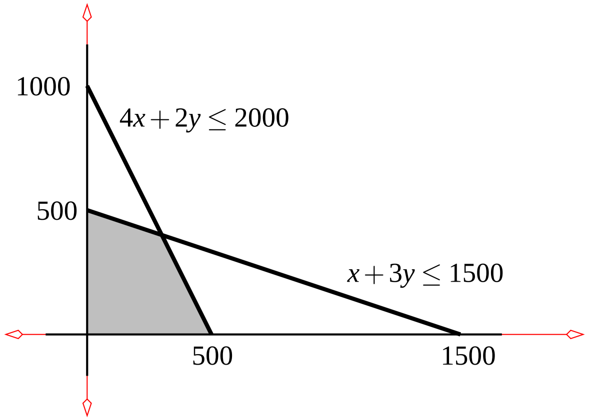
- Total press time is less than 1500 minutes per week, so

$$x + 3y \leq 1500$$

Plot the feasible region

- We need to know which x and y values obey all the constraints.





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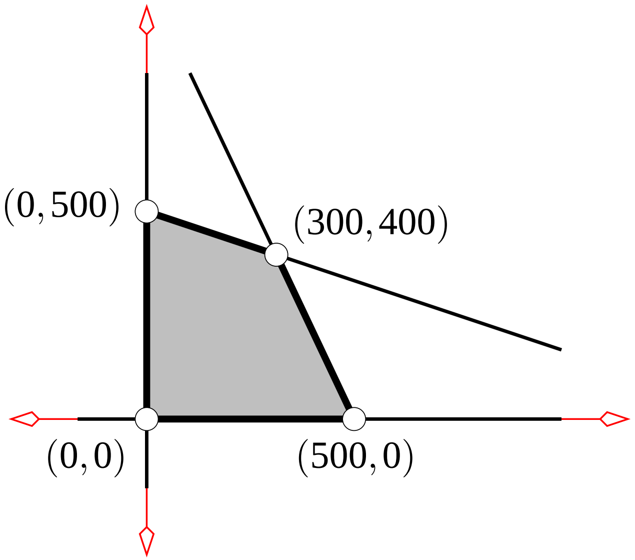
- Work out the corners

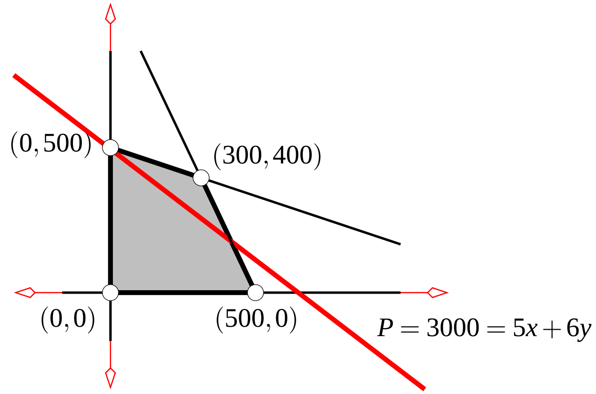


Plot lines of constant profit

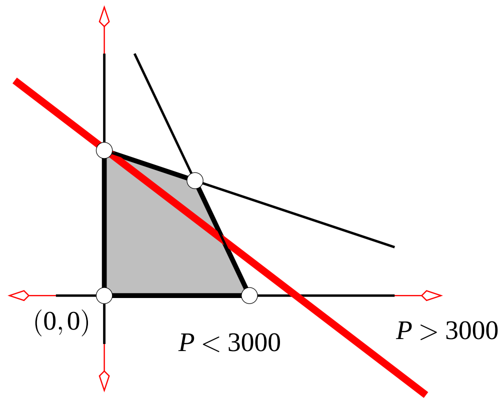
- Work at profit at one of the corners.

$$P = 5x + 6y \qquad \text{at } (x, y) = (0, 500) \text{ profit is } P = 3000$$

- Plot the line of constant profit

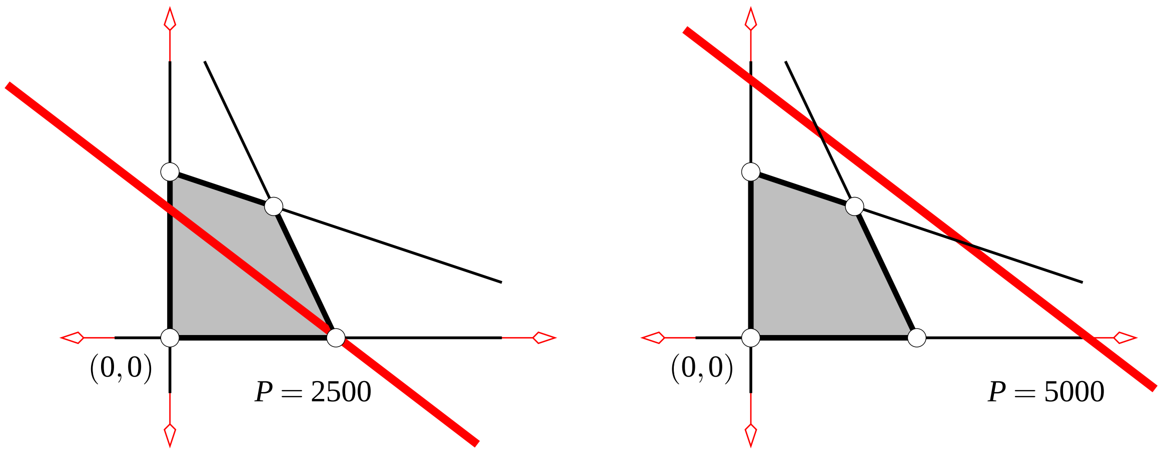


- Which way does P increase? Use halfplane test.



Where to test next?

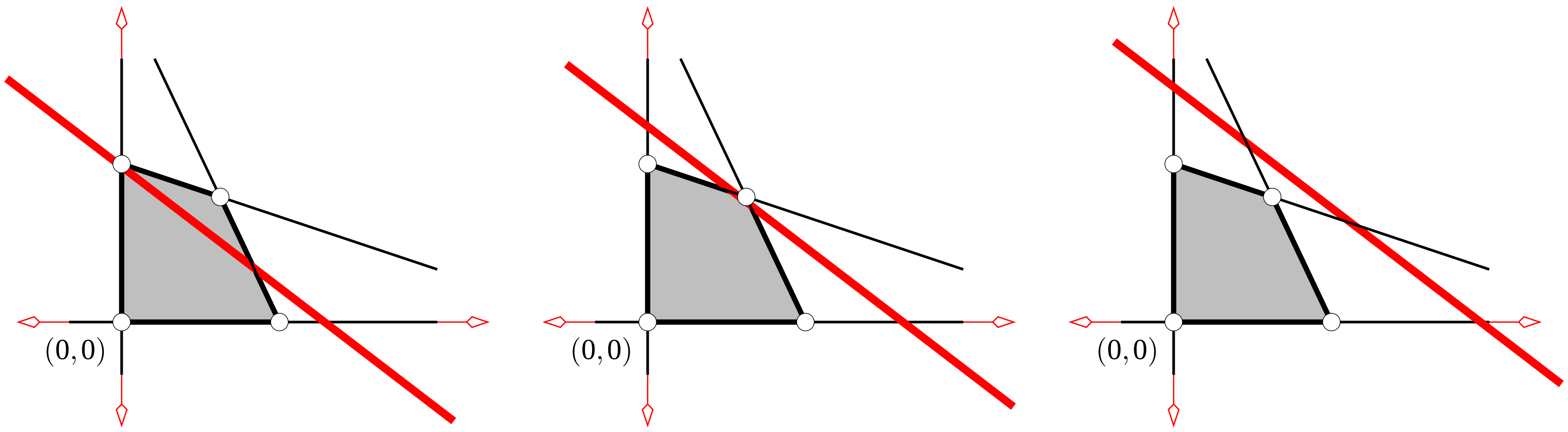
- Profit is less than 3000 towards bottom-left.
- Profit is greater than 3000 towards top-right.
- Increasing P moves the line to the top-right.



- As we move the line to the top-right we keep increasing P.
- If there is a point in the feasible region on the line then we can make that profit.
- If there is no point in the feasible region on the line then we cannot make that profit.

So where is the maximum profit?

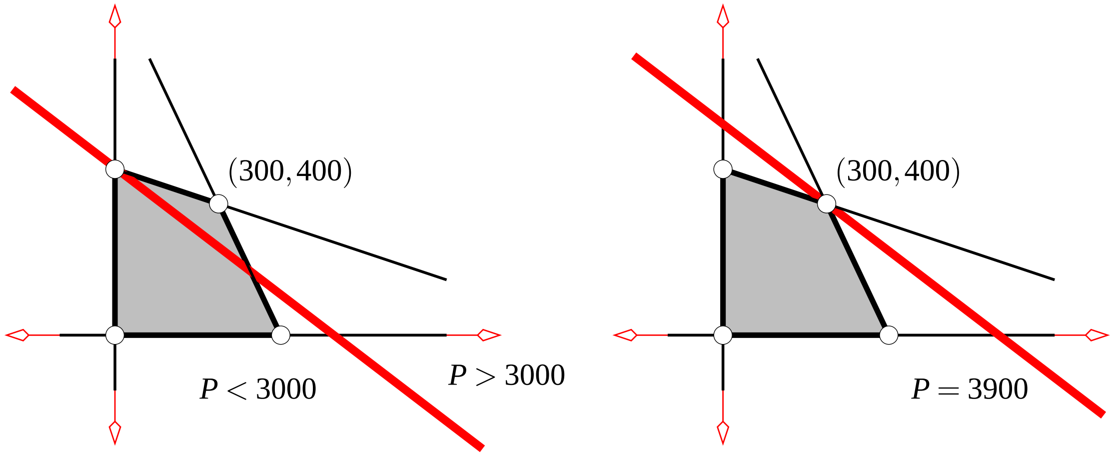
- As we move the line to the top-right profit increases.
- We need the line to touch a point in the feasible region.
- Stop moving the line when it is about to leave the region.
- That is the maximum profit.
- The line will be touching a corner — or maybe an edge.



- So maximum (or minimum) is always found at the corner of the feasible region.
- Sometimes an edge.

The answer

- We tested $(0, 500)$ and the profit was 3000.
- All the points below-left were lower profit.



- Only point left to test is $(300, 400)$ — profit is 3900.
- All points above-right of this line are bigger profit, but outside feasible region.
- All points below-left of this line are smaller profit.
- This is the maximum profit.
- 300 shirts and 400 jackets make 3900 per week profit and obey all constraints.