1. Let $MI$ be the event that a patient has had a myocardial infarct (a heart attack), and $S$ be the event that a patient has had a stroke. Then $P(S|MI) = 0.2$, $P(S|MI^c) = 0.35$, and $P(MI) = 0.4$. Therefore, using the Law of Total Probability applied to the partition \{MI, MI^c\},

$$P(S) = P(S|MI)P(MI) + P(S|MI^c)P(MI^c)$$

$$= 0.2 \times 0.4 + 0.35 \times 0.6$$

$$= 0.29$$

2. Let $G$ be the event that Susan is guilty. Note that in this question we have an initial value of $P(G) = 0.65$ (based on the judge’s opinion). We calculate a revised value based on the additional information that Robert and Julie give conflicting testimony (event $C$). We have that

$$P(C) = P(C|G)P(G) + P(C|G^c)P(G^c)$$

$$= P(\text{Robert lies}|G) \times 0.65 + P(\text{Julie lies}|G^c) \times 0.35$$

$$= 0.25 \times 0.65 + 0.3 \times 0.35$$

$$= 0.2675.$$

and

$$P(G \cap C) = P(C|G)P(G) = 0.25 \times 0.65 = 0.1625.$$

Therefore,

$$P(G|C) = \frac{P(G \cap C)}{P(C)}$$

$$= \frac{0.1625}{0.2675}$$

$$= 0.6075.$$

3. $$P(\text{at least one of next 6 buy black shoes}) = 1 - P(\text{none buy black shoes})$$

$$= 1 - (0.45)^6$$

$$= 0.9917.$$
4. (a) Let $C = \{\text{claim in 2003}\}$, $M = \{\text{male selected}\}$, and $M^c = \{\text{female selected}\}$. Then by the Law of Total Probability

\[
P(C) = P(C|M)P(M) + P(C|M^c)P(M^c)
\]

\[
= 0.6\alpha + 0.4\beta.
\]

(b) Now let $D = \{\text{claim in 2003} \cap \text{claim in 2004}\}$. By independence, $P(D|M) = \alpha \times \alpha = \alpha^2$. Similarly, $P(D|M^c) = \beta^2$. Replacing $C$ with $D$, and substituting $\alpha^2$ for $\alpha$ and $\beta^2$ for $\beta$ in (a) above, we obtain

\[
P(D) = 0.6\alpha^2 + 0.4\beta^2.
\]