1. (a) \( P(X \leq Y) = p(1, 1) + p(1, 2) = \frac{1}{7} + \frac{3}{7} = \frac{4}{7} \).

(b) \( X \) and \( Y \) are not independent since \( p(1, 1) = \frac{1}{7} \), \( p_X(1) = \frac{3}{7} \), and \( p_Y(1) = \frac{5}{7} \), and \( p(1, 1) \neq p_X(1)p_Y(1) \).

2. (a) \( P(X = 3, Y = 2) = P(Y = 2|X = 3)P(X = 3) = \binom{3}{2} \times 0.6^2 \times 0.4 \times 0.25 = 0.108. \)

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>( p_Y(y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y 1</td>
<td>0.15</td>
<td>0.25</td>
<td>0.4</td>
<td>0.35</td>
<td>0.43 \times 0.25</td>
</tr>
<tr>
<td>Y 2</td>
<td>0.15</td>
<td>0.6</td>
<td>2</td>
<td>0.6</td>
<td>3 \times 0.6 \times 0.4^2 \times 0.25</td>
</tr>
<tr>
<td>Y 3</td>
<td>0.15</td>
<td>0</td>
<td>0</td>
<td>0.6</td>
<td>3 \times 0.6^2 \times 0.25</td>
</tr>
<tr>
<td>( p_X(x) )</td>
<td>0.15</td>
<td>0.25</td>
<td>0.35</td>
<td>0.25</td>
<td>1</td>
</tr>
</tbody>
</table>

3. (a) For \( 0 \leq x \leq 1 \), \( f_X(x) = \int_0^1 4xydy = 2x \), 0, otherwise. By symmetry, for \( 0 \leq y \leq 1 \), \( f_Y(y) = 2y \), 0, otherwise.

(b) \( X \) and \( Y \) are independent since, for \( 0 \leq x, y \leq 1 \), \( f_{(X,Y)}(x,y) = f_X(x)f_Y(y) \).

(c) \[
P(X + Y < 1) = \int_0^1 \int_0^{1-x} 4xydydx
= \int_0^1 [2xy^2]_0^{1-x} dx
= \int_0^1 2x(1-x)^2dx
= \left[ x^2 - \frac{4}{3}x^3 + \frac{1}{2}x^4 \right]_0^1
= \frac{1}{6}.
\]
4. Refer to the figure below.

(a) For $0 \leq x \leq 1$, $f_X(x) = \int_0^y 8xydy = [4xy^2]_0^x = 4x^3$, 0, otherwise.

For $0 \leq y \leq 1$, $f_Y(y) = \int_y^1 8xydx = [4x^2y]_y^1 = 4y - 4y^3$, 0, otherwise.

(b) Refer to the figure below.

\[ P(X \leq 1/2 | Y \geq 1/4) = \frac{P(X \leq 1/2, Y \geq 1/4)}{P(Y \geq 1/4)} \]
\[ = \frac{\int_{1/4}^{1/2} \int_{x/4}^x 8xydydx}{\int_{1/4}^{1/2} (4y - 4y^3)dy} \]
\[ = \frac{\int_{1/4}^{1/2} 8x^2y dy}{\int_{1/4}^{1/2} (4y - 4y^3)dy} \]
\[ = \frac{9/256}{225/256} \]
\[ = \frac{9}{225}. \]

(c) We have that

\[ P(X \leq 1/2 | Y = 1/4) = \int_0^{1/2} f_{X|Y}(x|Y = 1/4)dx. \]

Now, for $\frac{1}{4} \leq x \leq 1$,

\[ f_{X|Y}(x|Y = 1/4) = \frac{f_{(X,Y)}(x, y)(x, \frac{1}{4})}{f_Y(\frac{1}{4})} \]
\[ = \frac{8x \times \frac{1}{4}}{4 \times \frac{1}{4} - 4 \times (\frac{1}{4})^3} \]
\[ = \frac{32x}{15}. \]
Therefore,

\[ P(X \leq 1/2|Y = 1/4) = \int_{1/4}^{1/2} \frac{32x}{15} \, dx \]

\[ = \left[ \frac{16}{15} x^2 \right]_{1/4}^{1/2} \]

\[ = \frac{16}{15} \times \left( \frac{1}{4} - \frac{1}{16} \right) \]

\[ = \frac{1}{5}. \]