620-202 Tutorial 7.

1. In a clinical trial, let the probability of a successful outcome have a prior distribution that is uniform over $[0, 1]$. Suppose that the first patient has a successful outcome. Find the Bayes estimate of θ that would be obtained for the squared error loss.

2. (7.2-1) Let Y be the sum of the observations from a Poisson distribution with mean θ. Let the prior p.d.f. of θ be gamma with parameters α and β so that

$$f(\theta) = \frac{1}{\Gamma(\alpha)\beta^\alpha}\theta^{\alpha-1}e^{-\theta/\beta}, 0 \leq \theta < \infty$$

   (a) Find the posterior p.d.f. of θ given $Y = y$. (Hint: You should be able to recognise the form of the numerator so only consider the terms that involve θ).

   (b) If the loss function is $[w(y) - \theta]^2$ find the Bayesian point estimate $w(y)$ of θ.

   (c) Show that this $w(y)$ is a weighted average of the maximum likelihood estimate y/n and the prior mean $\alpha\beta$, with respective weights $n/(n + 1/\beta)$ and $(1/\beta)(n + 1/\beta)$.

3. (7.2-4) Consider a random sample $X_1, \ldots, X_n$ from a distribution with p.d.f.

$$f(x|\theta) = 3\theta x^2 e^{-\theta x^3}, \quad 0 < x < \infty$$

   Let θ have a prior p.d.f. which is gamma with $\alpha = 4$ and $\beta = 1/4$. Find the conditional mean of θ, given $X_1 = x_1, \ldots, X_n = x_n$.