

620-202 Tutorial/Computing Laboratory 9.

- (8.2-7) Vitamin B_6 is one of the vitamins in a multiple vitamin pill manufactured by a pharmaceutical company. The pills are produced with a mean of 50 milligrams of vitamin B_6 per pill. The company believes there is a deterioration of 1 milligram per month, so that after 3 months they expect that $\mu = 47$. A consumer group suspects that $\mu < 47$ after 3 months.
 - Define a critical region to test $H_0 : \mu = 47$ against $H_1 : \mu < 47$ at the $\alpha = 0.05$ significance level based on a random sample of size $n = 20$. ($t_{0.05}(19) = 1.729$, $t_{0.05}(20) = 1.724$, $t_{0.025}(19) = 2.093$, $t_{0.025}(20) = 2.086$).
 - If the 20 pills yielded a mean of $\bar{x} = 46.94$ with standard deviation of $s = 0.15$, what is your conclusion?
 - What is the approximate p-value of this test?
- (8.2-6) Let X be the forced vital capacity (FVC) in liters for a female college student. Assume that $X \sim N(\mu, \sigma^2)$ approximately. Suppose it is known that $\mu = 3.4$ litres. A volleyball coach claims the FVC of volleyball players is greater than 3.4. She plans to test this using a random sample of size $n = 9$.
 - Define the null hypothesis.
 - Define the alternative hypothesis.
 - Define a critical region for which $\alpha = 0.05$. Illustrate this on a figure. ($t_{0.025}(8) = 2.306$, $t_{0.05}(8) = 1.859$, $t_{0.01}(8) = 2.896$)
 - Calculate the value of the test statistic if $\bar{x} = 3.556$ and $s = 0.167$.
 - What is your conclusion?
 - What is the approximate p-value of this test?
- (8.2-12) In May the fill weights of 6-pound boxes of laundry soap had a mean of 6.13 pounds with a standard deviation of 0.095. The goal was to decrease

the standard deviation. The company decided to adjust the filling machines and then test $H_0 : \sigma = 0.095$ against $H_1 : \sigma < 0.095$. In June a random sample of size $n = 20$ yielded $\bar{x} = 6.10$ and $s = 0.065$.

(a) At an $\alpha = 0.05$ level of significance was the company successful?

$$(\chi_{0.95}^2(19) = 10.117, \chi_{0.975}^2(19) = 8.906, \chi_{0.05}^2(19) = 30.143, \chi_{0.025}^2(19) = 32.852)$$

(b) What is the approximate p-value of your test?

4. (8.3-4) Among the data collected for the World Health Organisation air quality monitoring project is a measure of suspended particles in $\mu g/m^3$. Let X and Y equal the concentration of suspended particles in the city centres of Melbourne and Houston. Using $n = 13$ observations of X and $m = 16$ observations of Y , we shall test $H_0 : \mu_X = \mu_Y$ against $H_1 : \mu_X < \mu_Y$.

(a) Define the test statistic and the critical region assuming the variances are equal. Let $\alpha = 0.05$

(b) If $\bar{x} = 72.9$, $s_x = 25.6$, $\bar{y} = 81.7$ and $s_y = 28.3$, calculate the value of the test statistic and state your conclusion.

$$(t_{0.025}(27) = 2.052, t_{0.05}(27) = 1.703, (t_{0.1}(27) = 1.314, (t_{0.25}(27) = 0.684)$$

(c) Give limits for the p-value of this test.

(d) Test whether the assumption of equal variances is valid. Let $\alpha = 0.05$.

$$(F_{0.025}(12, 15) = 2.96, F_{0.975}(12, 15) = 0.314)$$

5. (8.2-10) R question. A company that manufactures brackets for an auto maker regularly selects brackets from the production line and performs a torque test. The goal is for the mean torque to equal 125. Let $X \sim N(\mu, \sigma^2)$ be the torque and suppose we take a random sample of size $n = 15$ to test $H_0 : \mu = 125$ against a two sided alternative. Suppose the following data are observed:

128.00	149.00	136.00	114.00	126.00	142.00	124.00	136.00
122.00	118.00	122.00	129.00	118.00	122.00	129.00	

Use the `t.test` command to test the hypotheses and construct a 95% confidence interval for μ .

6. (8.3-3) Let X be the weight in grams of a Low Fat Strawberry Kudo and Y the weight of a Low Fat Blueberry Kudo. Assume $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$. A random sample of 9 observations on X yielded

21.7, 21.0, 21.2, 20.7, 20.4, 21.9, 20.2, 21.6, 20.6

and a random sample of 13 observations on Y yielded

21.5, 20.5, 20.3, 21.6, 21.7, 21.3, 23.0, 21.3, 18.9, 20.0, 20.4, 20.8, 20.3

- (a) Test $H_0 : \sigma_X^2/\sigma_Y^2 = 1$ against a two-sided alternative. You may choose the significance level. (Hint: `var.test`)
- (b) Test $H_0 : \mu_X = \mu_Y$ against a two-sided alternative. You may choose the significance level. Your answer to (a) may guide your choice of test.
- (c) Construct a box plot to support your conclusions.
7. R question. Let X_1, \dots, X_{10} be a random sample of size 10 from a normal distribution, $N(\mu, \sigma^2)$. We shall test the null hypothesis $H_0 : \sigma^2 = 47.5$ against the alternative $H_1 : \sigma^2 < 47.5$.

- (a) Find a constant c so that $C = \{s^2 : s^2 \leq c\}$ is a critical region of size $\alpha = 0.05$. (Hint: use `qchisq`)
- (b) Define the power function $K(\sigma^2)$ for this test using `pchisq`.
- (c) Graph the power function for $2.5 \leq \sigma^2 \leq 47.5$.

```
> f = function(s2){pchisq(157.9429/s2,9)}
> s2 <- seq(2.5,47.5,.5)
> P <- f(s2)
> plot(s2,P,type="l")
```

- (d) For each of $\sigma^2 = 2.5, 7.5, 12.5, \dots, 47.5$, generate $m = 100$ random samples of size $n = 10$ from the normal distribution, $N(0, \sigma^2)$. For each value of σ^2 determine the number of times that H_0 was rejected. This can be done by writing a function and a loop. You should check you understand each step.

```
f2=function(s2){ X = matrix(rnorm(1000,0,sqrt(s2)),ncol=10);  
Y=apply(X,1,var);sum(Y<17.54921)};  
EP = rep(0,19)  
S2 = seq(2.5,47.5,2.5);  
for(k in 1:19)EP[k]=f2(S2[k]);  
EP = EP/100;
```

- (e) Show that the theoretical and empirical power functions are the same by plotting them on the same figure.

```
P = f(S2);  
matplot(S2,cbind(P,EP),type="l");
```

620-202 Tutorial/Computing Laboratory 9: Answers

1. (a) Critical region is:

$$t = \frac{\bar{x} - 47}{s/\sqrt{20}} < -t_{0.05}(19) = -1.729$$

(b) $t = (46.94 - 47)/0.15/\sqrt{20} = -1.789$. This is less than -1.729 so we reject H_0 .

(c) The p-value is between 0.025 and 0.05.

2. (a) $H_0 : \mu = 3.4$

(b) $H_1 : \mu > 3.4$

(c) $t = (\bar{x} - 3.4)/(s/3) > 1.859$

(d) $t = (3.556 - 3.4)/(0.167/3) = 2.802$

(e) Reject H_0 .

(f) $2.306 < 2.802 < 2.896$ so the p-value is between 0.01 and 0.025.

3. (a) The critical region is

$$\chi^2 = \frac{19s^2}{(0.095)^2} \leq 10.117.$$

The observed value is

$$\chi^2 = \frac{19(0.065)^2}{(0.095)^2} = 8.895$$

so we reject H_0 and there is evidence that the company is successful.

(b) As $\chi_{0.975}^2(19) = 8.906$, the approximate p-value is 0.025.

4. (a) Critical region is given by

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{12s_x^2 + 15s_y^2}{27} \left(\frac{1}{13} + \frac{1}{16} \right)}} \leq t_{0.05}(27) = -1.703$$

(b) Observed value is

$$t = \frac{72.9 - 81.7}{\sqrt{\frac{12 \times 25.6^2 + 15 \times 28.3^2}{27} \left(\frac{1}{13} + \frac{1}{16} \right)}} = -0.869 > -1.703$$

so we cannot reject H_0 .

(c) $0.10 < \text{p-value} < 0.25$.

(d)

$$\frac{s_x^2}{s_y^2} = \frac{25.6^2}{28.3^2} = 0.818 < 2.96 = F_{0.025}(12, 15)$$

so there is no evidence the variances differ.

5. `> t.test(X,mu=125)`

One Sample t-test

data: X

t = 1.0762, df = 14, p-value = 0.3000

alternative hypothesis: true mean is not equal to 125

95 percent confidence interval:

122.3522 132.9811

sample estimates:

mean of x

127.6667

p-value > 0.05 so we cannot reject H_0 .

6. (a) `> var.test(X,Y)`

F test to compare two variances

data: X and Y

F = 0.3624, num df = 8, denom df = 12, p-value = 0.1573

alternative hypothesis: true ratio of variances is not equal to 1

95 percent confidence interval:

0.1031926 1.5219149

sample estimates:

ratio of variances

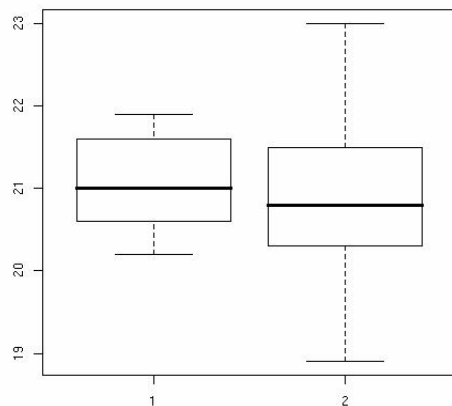
0.3623894

so we cannot reject H_0 at the 5% level of significance.

```
(b) > t.test(X,Y,mu=0,var.equal=T)
Two Sample t-test
data: X and Y
t = 0.3742, df = 20, p-value = 0.7122
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.6451712  0.9272225
sample estimates:
mean of x mean of y
 21.03333  20.89231
```

so do not reject H_0 at the 5% level of significance.

(c) A boxplot is



7. (a) The critical region is

$$\frac{(n-1)s^2}{\sigma^2} < \chi_{0.95}^2(9)$$

so reject H_0 if

$$s^2 < \frac{\chi_{0.95}^2(9)\sigma^2}{n-1} = \frac{3.325113 \times 47.5}{9} = 17.54921$$

(b) Power is

$$\begin{aligned} K(\sigma^2) &= P(s^2 < 17.54921 \mid \sigma^2) = P\left(\frac{(n-1)s^2}{\sigma^2} < \frac{9 \times 17.54921}{\sigma^2} \mid \sigma^2\right) \\ &= P(\chi^2(9) < \frac{157.9429}{\sigma^2} \mid \sigma^2) \end{aligned}$$

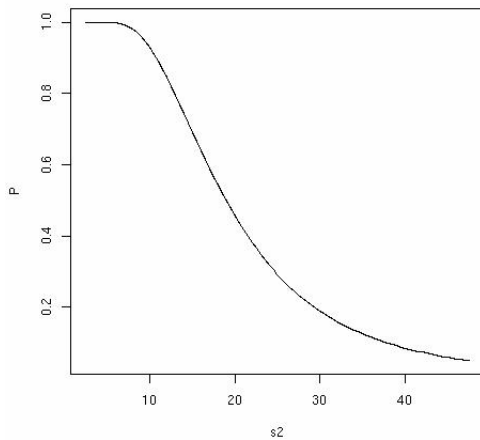
Define this in R and test at H_0

```
> f = function(s2){pchisq(157.9429/s2,9)}  
> f(47.5)  
[1] 0.05000004
```

(c) `> s2 <- seq(2.5,47.5,.5)`

```
> P <- f(s2)
```

```
> plot(s2,P,type="l")
```



(d) no output required.

(e) power plots are

