1. Consider the function \( f(x, y) = \frac{3x^5 - y^4}{x^4 + 4y^4} \).

(a) Find \( \lim_{(x,y) \to (0,0)} f(x, y) \), if it exists.

(b) Where is \( f(x, y) \) continuous?

In each part, explain your answer briefly. [7 marks]

2. Determine the second order Taylor polynomial for \( f(x, y) = \cos(x + 2y + y^2) \) about the origin. [8 marks]

3. Find the length of the path

\[ c(t) = t\mathbf{i} - t^2\sqrt{\frac{3}{2}}\mathbf{j} + t^3\mathbf{k} \quad \text{for} \quad 0 \leq t \leq 3. \]

[5 marks]

4. Consider the vector field \( \mathbf{V} = (x + 3y^2)\mathbf{i} + (y - 2z)\mathbf{j} + (x^3 + \lambda z)\mathbf{k} \) where \( \lambda \) is a real constant.

(a) Is \( \mathbf{V} \) an irrotational vector field? Explain briefly.

(b) Find the constant \( \lambda \) so that \( \mathbf{V} \) is an incompressible vector field.

(c) If \( \lambda \) is the value found in (b), give a physical interpretation of your answers to (a) and (b). Use diagrams to illustrate your answers. [14 marks]

5. Is the path \( c(t) = \left( \frac{1}{t^3}, e^t, \frac{1}{t} \right) \) a flow line of the vector field \( \mathbf{F} = (-3z^4, y, -z^2) \) for \( t > 0 \)? Explain briefly. [4 marks]

6. Let \( R \) be the region enclosed by \( x = 1 - y^2 \) and \( y = -x - 1 \).

(a) Sketch the region \( R \).

(b) Using double integrals, find the area of \( R \). [12 marks]