

620–231 VECTOR ANALYSIS

SAMPLE MID SEMESTER TEST

1. Consider the function $f(x, y) = \frac{3x^5 - y^4}{x^4 + 4y^4}$.

(a) Find $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$, if it exists.

(b) Where is $f(x, y)$ continuous?

In each part, explain your answer briefly.

[7 marks]

2. Determine the second order Taylor polynomial for $f(x, y) = \cos(x + 2y + y^2)$ about the origin.

[8 marks]

3. Find the length of the path

$$\mathbf{c}(t) = t\mathbf{i} - t^2\sqrt{\frac{3}{2}}\mathbf{j} + t^3\mathbf{k} \quad \text{for } 0 \leq t \leq 3.$$

[5 marks]

4. Consider the vector field $\mathbf{V} = (x + 3y^2)\mathbf{i} + (y - 2z)\mathbf{j} + (x^3 + \lambda z)\mathbf{k}$ where λ is a real constant.

(a) Is \mathbf{V} an irrotational vector field? Explain briefly.

(b) Find the constant λ so that \mathbf{V} is an incompressible vector field.

(c) If λ is the value found in (b), give a physical interpretation of your answers to (a) and (b). Use diagrams to illustrate your answers.

[14 marks]

5. Is the path $\mathbf{c}(t) = \left(\frac{1}{t^3}, e^t, \frac{1}{t}\right)$ a flow line of the vector field $\mathbf{F} = (-3z^4, y, -z^2)$ for $t > 0$? Explain briefly.

[4 marks]

6. Let R be the region enclosed by $x = 1 - y^2$ and $y = -x - 1$.

(a) Sketch the region R .

(b) Using double integrals, find the area of R .

[12 marks]