

620–231 VECTOR ANALYSIS

SAMPLE MID SEMESTER TEST

ANSWERS

1. (a) The function $f(x, y)$ has a different limit when approaching the origin along $x = 0$ and $y = 0$, so the limit does not exist.

(b) $f(x, y)$ is continuous for all (x, y) except at the origin.

2. $p_2(x, y) = 1 - \frac{1}{2}x^2 - 2xy - 2y^2$

3. $\text{arclength} = \int_0^3 \sqrt{1 + 6t^2 + 9t^4} dt = \int_0^3 1 + 3t^2 dt = 30$ units.

4. (a) \mathbf{V} is NOT an irrotational vector field since $\nabla \times \mathbf{V} \neq \mathbf{0}$.

(b) \mathbf{V} is an incompressible vector field if $\nabla \cdot \mathbf{V} = 0$ so $\lambda = -2$.

(c) See lecture notes for physical explanations of div and curl.

5. $\mathbf{c}(t)$ is a flow line of \mathbf{F} since $\mathbf{F}[\mathbf{c}(t)] = \mathbf{c}'(t)$.

6. Using horizontal strips, the area is $\int_{-1}^2 \int_{-1-y}^{1-y^2} dx dy = \frac{9}{2}$ square units.