1. (a) Find the three constants $a, b, c$ so that the vector field
\[ \mathbf{V} = (x + 2y + az, bx - 3y - z, 4x + cy + 2z) \] is an irrotational vector field and hence find a function $f(x, y, z)$ such that $\mathbf{V} = \nabla f$.

(b) If $\mathbf{r} = (x, y, z)$, $r = |\mathbf{r}|$ and $\mathbf{F} = \frac{\mathbf{r}}{r^3}$ find $\nabla \cdot \mathbf{F}$.

2. Find the length of the curve $\mathbf{c}(t) = \left(2t^{3/2}, \cos(2t), \sin(2t)\right)$ for $0 \leq t \leq 1$.

3. Consider the double integral:
\[ \int_{0}^{4} \int_{\sqrt{y}}^{2} \exp(x^3) \, dx \, dy. \]
Sketch the domain of integration and hence change the order of integration to evaluate the integral.

4. Consider the triple integral:
\[ \int_{-2}^{2} \left( \int_{0}^{\sqrt{4-y^2}} \left( \int_{x^2+y^2-4}^{0} (x^2 + y^2 + z) \, dz \right) \, dx \right) \, dy. \]

(a) Sketch the region $R$ of integration.

(b) Draw a separate diagram of the $xy$, $xz$ and $yz$ coordinate planes and show the curves, and their equations, were the surface of $R$ intersects the corresponding plane. Do not evaluate the integral.

Note: Full working must be shown in your solutions. Marks will be deducted for incomplete working.