* θ - angle measured anticlockwise from +ve x axis (polar axis) to $\overrightarrow{OP}$

Note:

① If $r=0$, θ is arbitrary. So pole is given by $(0,\theta)$.

② θ is not unique.

③ To find $\theta(x,y)$ we restrict θ to $0 \le \theta < 2\pi$. Then

$$\theta = \arctan\left(\frac{y}{x}\right) \qquad x>0,\ y\ge 0$$

$$= \pi + \arctan\left(\frac{y}{x}\right) \qquad x<0$$

$$= 2\pi + \arctan\left(\frac{y}{x}\right) \qquad x>0,\ y<0$$

$$= \frac{\pi}{2} \qquad x=0,\ y>0$$

$$= \frac{3\pi}{2} \qquad x=0,\ y<0$$

where $-\frac{\pi}{2} < \arctan\left(\frac{y}{x}\right) < \frac{\pi}{2}$

eg 1. Convert $(r,\theta) = (2, \frac{\pi}{6})$ to cartesian coordinates.

eg 2. Convert $(x,y) = (-1,-1)$ to polar coordinates.

Special cases

* $r = a$ (constant)

* $\theta = \alpha$ (constant)