Case 1 : $\alpha_1 > \alpha_2 > 0$

Draw the phase portrait for the system $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

given $\alpha_1 = 3, \quad \alpha_2 = 1$ eigenvalues

$\underset{\sim}{v}_1 = \begin{bmatrix} 1 \\ +1 \end{bmatrix}, \quad \underset{\sim}{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\left\{\begin{matrix} \text{corresponding} \\ \text{eigenvectors} \end{matrix}\right\}$

$y = +x \qquad y = \pm x$

at the critical point

Case 2 : $\alpha_1 < \alpha_2 < 0$

Draw the phase portrait for the system $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -10 & -1 \\ 15 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

given $\alpha_1 = -7$, $\alpha_2 = -5$ eigenvalues

$\underset{\sim}{v}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $\underset{\sim}{v}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$ $\left\{ \begin{matrix} \text{corresponding} \\ \text{eigenvectors} \end{matrix} \right\}$

at the critical point

Case 3: $\alpha_1 > 0 > \alpha_2$

Draw the phase portrait for the system

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 5 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

given $\alpha_1 = 1$, $\alpha_2 = -1$ eigenvalues

$\underset{\sim}{v_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\underset{\sim}{v_2} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ $\left\{ \begin{matrix} \text{corresponding} \\ \text{eigenvectors} \end{matrix} \right\}$

at the critical point

Case 4 α_1 and α_2 are imaginary

$$\alpha_1 = i\eta \quad \alpha_2 = -i\eta$$

This case was done as a demonstration earlier and we found that the critical point was a stable centre and the trajectories were either ellipses or circles. Remember that you need to determine the direction of the flow lines (clockwise or anticlockwise) by finding $\dot{\underset{\sim}{x}} = \dot{x}\underset{\sim}{i} + \dot{y}\underset{\sim}{j}$ at any point [(1,0) is a good point!]

Case 5 α_1 and α_2 are complex conjugates and have non-zero real part.

As you will recall a typical solution is of the form
$$\begin{bmatrix} x \\ y \end{bmatrix} = e^{\mu t}\left(\cos\eta t \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + \sin\eta t \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}\right)$$

where $\mu, \eta, A_1, A_2, B_1, B_2 \in \mathbb{R}$.

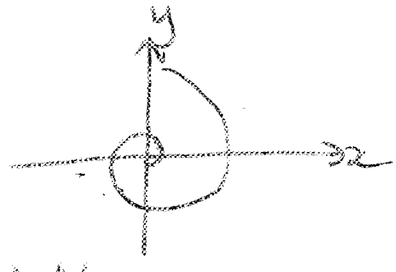
The presence of trig. functions indicates periodic winding about the critical point.

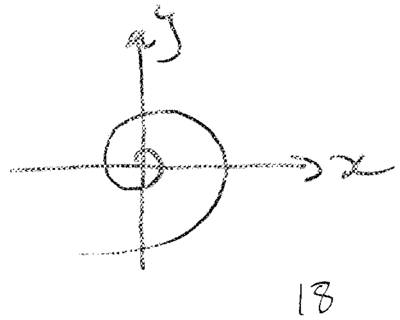
There are

2 spirals

[If $\mu > 0$ unstable

[If $\mu < 0$ asymptotically stable.





Draw the phase portrait for the system

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}-1 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$

(i) Critical pt @ $(\dot{x},\dot{y}) = (0,0)$ gives $(x,y)=(0,0)$

(ii) solving for the eigenvalues we get

$$\alpha = -1 \pm i$$

so a solution will give us no information about the phase portrait. Here is my solution anyway (you should check!!)

$$\begin{bmatrix}x\\ y\end{bmatrix} = ae^{-t}\begin{bmatrix}\cos t\\ -\sin t\end{bmatrix} + be^{-t}\begin{bmatrix}\sin t\\ \cos t\end{bmatrix}$$

(iii) The eigenvectors are trig functions and so the eigenvectors are winding too!

$\therefore$ can't draw

(iv) Velocity vectors

$\dot{x} = -x + y$

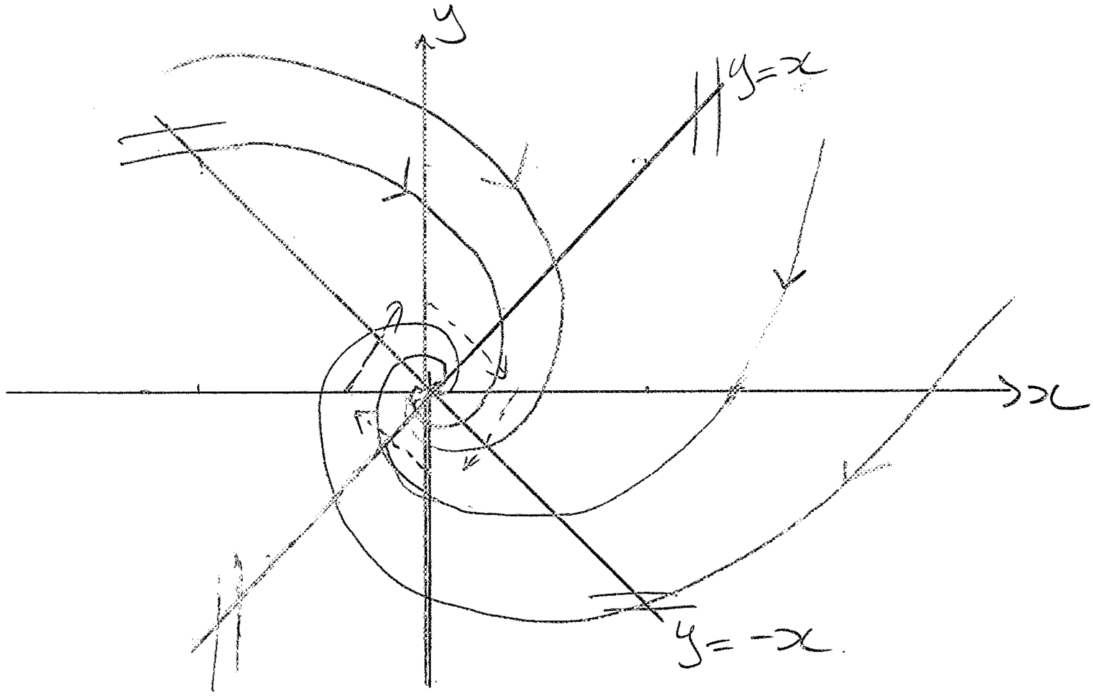
$\dot{y} = -x - y$

(x,y)	$(1,0)$	$(0,1)$	$(-1,0)$	$(0,-1)$
$\underset{\sim}{\dot{x}} = (\dot{x},\dot{y})$	$(-1,-1)$	$(1,-1)$	$(1,1)$	$(-1,1)$

(v) Nullclines HTPs $\dot{y} = 0$ $\therefore$ HTPs along $y = -x$

VTPs $\dot{x} = 0$ $\therefore$ VTPs along $y = x$

(vi) Irrelevant here



(vii) As $t \to \infty$ $e^{-t} \to 0$ $\therefore x, y \to 0$

$\therefore$ have an asymptotically stable spiral