Case 6 $\quad \alpha_1 = \alpha_2 \neq 0$

Here we can have 2 cases

(a) 2 distinct eigenvectors

or (b) A degenerate node results when there is only 1 eigenvector and we use the method of undetermined coefficients to generate our 2nd solution

Case 6(a) Draw the phase portrait for
$$\dot{x} = x$$
$$\dot{y} = y$$

Case 6(b) Draw the phase portrait for the system

$$\dot{x} = 4x + y$$
$$\dot{y} = -x + 2y$$

where the solution is of the form

$$\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{Ae^{3t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{y=-x} + Be^{3t}\left(t\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

Case 7: $\alpha_1 = 0$ has 2 subcases

(a) $\alpha_2 \neq 0$

(b) $\alpha_2 = 0$

Case 7(a): Draw the phase portrait

for
$$\dot{x} = x + y$$
$$\dot{y} = x + y$$

where $\lambda = 2$ $\underset{\sim}{V_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and

$\lambda = 0$ $\underset{\sim}{V_0} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

are the eigenvalue/eigenvector solutions

Case 7(b): Draw the phase portrait for the system $\dot{x} = y$
$\dot{y} = 0$

where a solution is of the form $\begin{bmatrix} x \\ y \end{bmatrix} = A \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{x\,\text{axis}} + B\left(t\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$