$$\therefore \frac{\partial^2 u}{\partial t^2} = -c\left(-c\frac{\partial^2 v}{\partial \xi^2} + c\frac{\partial^2 v}{\partial \xi \partial \eta}\right) + c\left(-c\frac{\partial^2 v}{\partial \eta \partial \xi} + c\frac{\partial^2 v}{\partial \eta^2}\right)$$

$$= c^2\left\{\frac{\partial^2 v}{\partial \xi^2} - 2\frac{\partial^2 v}{\partial \xi \partial \eta} + \frac{\partial^2 v}{\partial \eta^2}\right\} \quad (2)$$

Now $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ so substitute (1) and (2)

$$c^2\left\{\frac{\partial^2 v}{\partial \xi^2} - 2\frac{\partial^2 v}{\partial \xi \partial \eta} + \frac{\partial^2 v}{\partial \eta^2}\right\} = c^2\left\{\frac{\partial^2 v}{\partial \xi^2} + 2\frac{\partial^2 v}{\partial \xi \partial \eta} + \frac{\partial^2 v}{\partial \eta^2}\right\}$$

so $4c^2 \frac{\partial^2 v}{\partial \xi \partial \eta} = 0 \qquad 4c^2 \neq 0$

$$\Rightarrow \frac{\partial^2 v}{\partial \xi \partial \eta} = 0 \text{ or } \frac{\partial^2 v}{\partial \eta \partial \xi} = 0$$

$$\Rightarrow \frac{\partial}{\partial \xi}\left(\frac{\partial v}{\partial \eta}\right) = 0$$

$\therefore \frac{\partial v}{\partial \eta}$ is a constant wrt the ξ variable

$\Rightarrow \frac{\partial v}{\partial \eta} = g(\eta)$ arbitraty function of η

Integrate $v = \int g(\eta)\, d\eta$

$$= \underbrace{F(\xi)}_{\text{constant of integration wrt } \eta} + \int g(\eta)\, d\eta$$

$$v = \underbrace{F(\xi)}_{\uparrow} + \int g(\eta)\,d\eta$$

constant of integration wrt η

$\therefore$ an arbitrary function of ξ

$$\therefore v = F(\xi) + G(\eta)$$

$$\Rightarrow u(x,t) = F(x-ct) + G(x+ct)$$

where <u>F and G are arbitrary functions</u>

$\boxed{\text{Wave equation pde}}$

$\downarrow$

use <u>change of variable</u> $(x,t) \to (\xi, \eta)$ $\longrightarrow$ $\dfrac{\partial^2 v}{\partial \xi \partial \eta} = 0$

$\downarrow$

$$\boxed{\begin{array}{c} \dfrac{\partial v}{\partial \eta} = g(\eta) \\ \text{like ode} \end{array}}$$

later if we have time

For diffusion equation are there ways of converting to ode?

one way, for special initial (and boundary conditions) can introduce <u>change of variable</u>

Separation of Variables

- method of converting pde $\rightarrow$ ode
- conditions when we can use this method
- method taught by example in lecture