

## 620-261 Introduction to Operations Research

### ASSIGNMENT 1

*Post in boxes by 3.00 pm on Friday 10th March 2006*

1. Four jobs are to be processed on a printing machine. The machine can process only one job at a time. When a new job is processed, the machine must be adjusted and this set-up process is time consuming. The set-up times (in hours) involved in switching from job  $i$  to job  $j$  is given by the  $(i, j)$  entry of the table below, for example, the set-up time incurred in switching from job  $B$  to job  $D$  is equal to 5 hrs. There is no set-up time for the first job processed.

	A	B	C	D
A	–	3	1	4
B	2	–	1	5
C	4	3	–	4
D	2	2	2	–

The Manager is interested in a scheduling policy that will process the jobs in an order that will minimize the total set-up time (sum of the all the individual set-up times). Formulate a mathematical model for this problem.

#### **Solution**

Let  $x_{ij} = 1$  if job  $j$  is processed immediately after job  $i$  and zero otherwise. Then we can formulate the problem as

$$\min(3x_{12} + x_{13} + 4x_{14} + 2x_{21} + x_{23} + 5x_{24} + 4x_{31} + 3x_{32} + 4x_{34} + 2x_{41} + 2x_{42} + 2x_{43})$$

such that

$$\begin{aligned}\sum_{ij} x_{ij} &= 3 \\ \sum_j x_{ij} &\leq 1 \quad \forall i \\ \sum_i x_{ij} &\leq 1 \quad \forall j \\ x_{ij} + x_{ji} &\leq 1 \quad \forall i, j \\ x_{ij} &\in \{0, 1\} \quad \forall i, j.\end{aligned}$$

NB: Think about why we need all the constraints. The first three are reasonably obvious. The reason for the fourth one is subtle. The markers won't expect you to have thought of it.

2. A company assembles both cars and trucks. The assembly is carried out in three departments: sheet metal stamping, engine assembly and final assembly. The number of person hours required for assembling cars and trucks in each of these departments is shown below:

	sheet metal	engine assembly	final assembly
cars	50	50	25
trucks	25	50	100

Each department has a capacity of 200,000 person hours per year. Cars earn a profit of \$6000 each, and trucks \$10,000 each, for the company. Formulate a linear program that can be used to determine how many cars and trucks the company should manufacture each year to maximise its profit?

**Solution**

Let  $x_1$  be the number of cars manufactured each year and  $x_2$  be the number of trucks manufactured each year. Then we can formulate the problem as

$$\max 6,000x_1 + 10,000x_2$$

such that

$$\begin{aligned} 50x_1 + 25x_2 &\leq 200,000 \\ 50x_1 + 50x_2 &\leq 200,000 \\ 25x_1 + 100x_2 &\leq 200,000 \\ x_1, x_2 &\geq 0. \end{aligned}$$