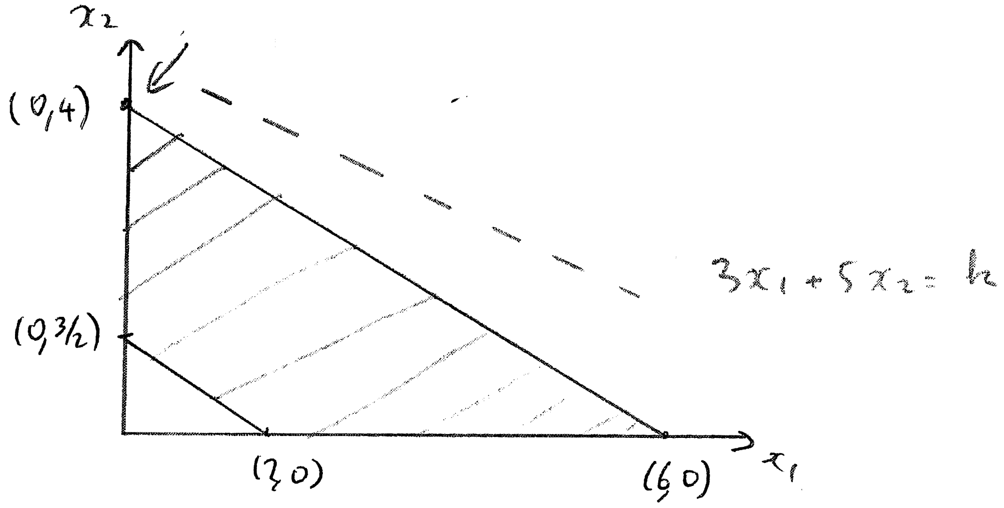
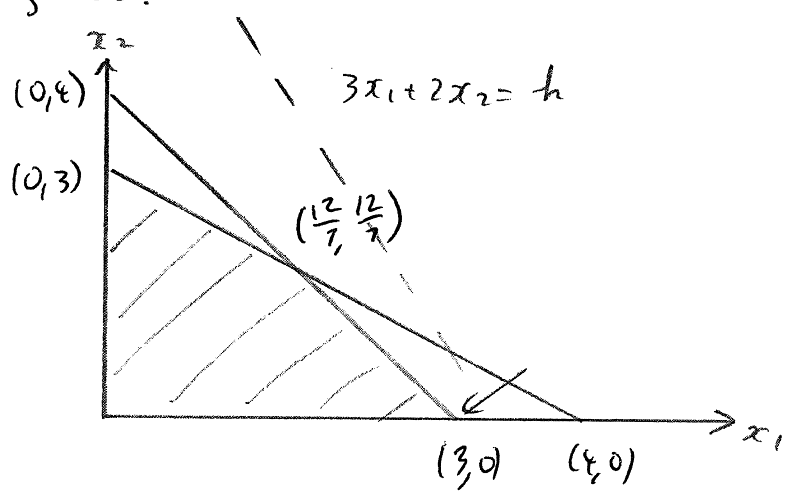
SOLUTIONS TO ASSIGNMENT 2

1/ (a)



The maximal value of the objective function is at (0,4) : $z=20$.

(b)



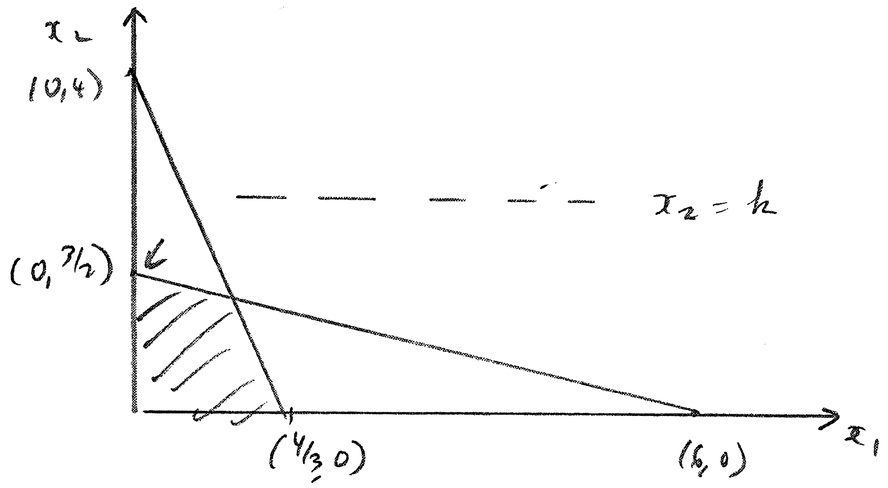
The maximal value of the objective function is at (3,0): $z=9$.

(c). First we need to transform the problem.
The first constraint gives $x_3 = 4-3x_1-x_2$.
$x_3 \geq 0 \Rightarrow 3x_1+x_2 \leq 4$. This dominates the first constraint. Therefore the problem is

$$\max \quad 3x_1+2x_2+x_3 = 3x_1+2x_2+4-3x_1-x_2 = x_2+4$$

(might as well be x_2).

such that.
$$3x_1+x_2 \leq 4$$
$$x_1+4x_2 \leq 6$$
$$x_1, x_2 \geq 0.$$



The maximal value occurs when $\underline{x} = (0, 3/2)$. Here $x_3 = 5/2$

and $z = 3 \cdot 0 + 2 \cdot (3/2) + 5/2 = 11/2$.

2. Let x_i be the number of litres of grade i whisky that the company choose to make. Then we want to

$$\max. \quad z = 12x_1 + 6x_2 + 4x_3$$
such that
$$\begin{aligned} .9x_1 + .3x_2 + .1x_3 &\le 3000 \\ .1x_1 + .5x_2 + .3x_3 &\le 5{,}000 \\ .2x_2 + .6x_3 &\le 10{,}000. \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

If you get this far, you can be happy with what you have done. Even for a small problem like this solving it using our current state of knowledge takes some effort. You could try a 3.D "graphical solution", or you could re-write the problem in canonical form and look for basic feasible solutions.

i.e. $\max \quad z = 12x_1 + 6x_2 + 4x_3$
such that
$$\begin{aligned} .9x_1 + .3x_2 + .1x_3 + x_4 &= 3{,}000 \\ .1x_1 + .5x_2 + .3x_3 + x_5 &= 5{,}000 \\ .2x_2 + .6x_3 + x_6 &= 10{,}000. \\ x_1, x_2, x_3, x_4, x_5, x_6 &\ge 0 \end{aligned}$$

There are $\binom{6}{3} = 20$ different combinations of 3 columns that we can choose from the six. This would take a

x_1	x_2	x_3	x_4	x_5	x_6			
0	0	0	3000	5000	10000	0	feasible	
0	0	30000	0	-4000	-8000	120000	not feasible	
0	0	16666.6667	1333.33333	0	0	66666.6667	feasible	
0	0	16666.6667	1333.33333	0	0	66666.6667	feasible	
0	10000	0	0	0	8000	60000	feasible	
0	10000	0	0	0	8000	60000	feasible	
0	50000	0	-12000	-20000	0	300000	not feasible	
0	10000	0	0	0	8000	60000	feasible	
0	5000	15000	0	-2000	0	90000	not feasible	
0	0	16666.6667	1333.33333	0	0	66666.6667	feasible	
3333.33333	0	0	0	4666.66667	10000	40000	feasible	
50000	0	0	-42000	0	10000	600000	not feasible	
1538.46154	0	16153.8462	0	0	307.692308	83076.9231	feasible	optimal
1481.48148	0	16666.6667	0	-148.148148	0	84444.4444	not feasible	
0	0	16666.6667	1333.33333	0	0	66666.6667	feasible	
0	10000	0	0	0	8000	60000	feasible	
-13333.3333	50000	0	0	-18666.6667	0	140000	not feasible	
-200000	50000	0	168000	0	0	-2100000	not feasible	
1600	-400	16800	0	0	0	84000	not feasible	

So the optimal solution is at $x_1 = 1538.46$, $x_2 = 0$, $x_3 = 16153.85$
where $z = 83{,}076.92$.
So the whisky maker should make 1538.46 litres of grade 1
0 litres of grade 2
& 16153.85 litres of grade 3
to get \$83,076.92 profit.