Assignment 7 Solutions

1/ $\max \quad 3x_1 + 2x_2 - x_4$

$$\begin{aligned} \text{st} \quad & 4x_1 + 3x_2 + x_3 = 10 \\ & 5x_1 - 6x_2 + x_3 \le 12 \\ & 6x_1 + 7x_2 \ge 11. \end{aligned}$$

$x_1 \ge 0, \; x_3 \ge 0, \; x_4 \ge 0, \; x_2$ urs.

is the same as

$\max \quad 3x_1 + 2x_2 - x_4$

$$\begin{aligned} \text{st} \quad & 4x_1 + 3x_2 + x_3 \le 10 \\ & -4x_1 - 3x_2 - x_3 \le -10 \\ & 5x_1 - 6x_2 + x_3 \le 12 \\ & -6x_1 - 7x_2 \le -11 \end{aligned}$$

$x_1 \ge 0, \; x_3 \ge 0, \; x_4 \ge 0, \; x_2$ urs

which is the same as.

$\max \quad 3x_1 + 2x_2^{(1)} - 2x_2^{(2)} - x_4$

$$\begin{aligned} \text{st} \quad & 4x_1 + 3x_2^{(1)} - 3x_2^{(2)} + x_3 \le 10 \\ & -4x_1 - 3x_2^{(1)} + 3x_2^{(2)} - x_3 \le -10 \\ & 5x_1 - 6x_2^{(1)} + 6x_2^{(2)} + x_3 \le 12 \\ & -6x_1 - 7x_2^{(1)} + 7x_2^{(2)} \le -11 \end{aligned}$$

$x_1, x_2^{(1)}, x_2^{(2)}, x_3, x_4 \ge 0.$

The dual of this is.

$\min \quad 10y_1^{(1)} - 10y_1^{(2)} + 12y_2 - 11y_3$

$$\begin{aligned} \text{st} \quad & 4y_1^{(1)} - 4y_1^{(2)} + 5y_2 - 6y_3 \ge 3. \\ & 3y_1^{(1)} - 3y_1^{(2)} + 6y_2 - 7y_3 \ge 2 \\ & -3y_1^{(1)} + 3y_1^{(2)} + 6y_2 + 7y_3 \ge -2. \\ & y_1^{(1)} - y_2^{(2)} + y_2 \ge 0 \\ & 0y_1^{(1)} - 0y_1^{(2)} + 0y_2 - 0y_3 \ge -1. \leftarrow \text{redundant.} \end{aligned}$$

$y_1^{(1)}, y_1^{(2)}, y_2, y_3 \ge 0$

Putting $y_1^* = y_1^{(1)} - y_2^{(1)}$ and combining the second and third constraints, this is

$$
\begin{aligned}
4y_1 + 5y_2 - 6y_3 &\geq 3 \\
3y_1 - 6y_2 - 7y_3 &= 2 \\
y_1 + y_2 &\geq 0
\end{aligned}
$$

y_1 urs, $y_2 \geq 0$, $y_3 \geq 0$.

2/ (i) D is min $w = 14y_1 + 4y_2$

st
$$
\begin{aligned}
y_1 + 3y_2 &\geq 1 \\
5y_1 + 2y_2 &\geq 4 \\
4y_1 + y_2 &\geq 3 \\
y_1, y_2 &\geq 0.
\end{aligned}
$$

(ii)

BV	z	x_1	x_2	x_3	x_4	x_5	RHS	
x_4	0	1	5	4	1	0	14	$R_1 = R_1 - 5R_2'$
x_5	0	3	$\boxed{2}$	1	0	1	4	$R_2' = R_2/2$
z	1	-1	-4	-3	0	0	0.	$R_3 = R_3 + 4R_2'$
x_4	0	$-\frac{13}{2}$	0	$\boxed{3/2}$	1	$-5/2$	4	$R_1' = \frac{2}{3}R_1$
x_2	0	$3/2$	1	$1/2$	0	$1/2$	2.	$R_2 = R_2 - \frac{1}{2}R_1'$
z	1	5	0	-1	0	2	8	$R_3 = R_3 + R_1'$
x_3	0	$-\frac{13}{3}$	0	1	$\frac{2}{3}$	$-\frac{5}{3}$	$\frac{8}{3}$.	
x_2	0	$\frac{11}{3}$	1	0	$-\frac{1}{3}$	$\frac{4}{3}$.	$\frac{2}{3}$.	
z	1	$\frac{2}{3}$	0	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{32}{3}$.	

(iii) The optimal solution to the dual is

$y_1 = 2/3,\ y_2 = 1/3 \quad w = 32/3$

3/ (i) D is min $w = 6y_1 - 3y_2$

st $2y_1 - 3y_2 \geq 1$

$y_1 - 2y_2 \geq -1$

$5y_1 + 4y_2 \geq 3$

$y_1 \geq 0 \quad y_2 \geq 0$

NB. The primal we solve has $3x_1 + 2x_2 - 4x_3 \geq 3$ and corresponding dual variable $\bar{y}_2 \leq 0$. Thus $\bar{y}_2$ is the negative of our original y_2. By reading the reduced cost corresponding to x_6 off the tableau, we see that $\bar{y}_2 = -1/23$, which gives us $y_2 = 1/23$.

(ii) We need to multiply the second constraint by -1, then introduce surplus and artificial variables to solve the primal via the simplex algorithm

BV	w/z	x_1	x_2	x_3	x_4	x_5	x_6	RHS	
x_4	0	2	1	5	1	0	0	6	$R_1 = R_1 - 2R_2'$
$\underline{x_6}$	0	$\boxed{3}$	2	-4	0	-1	1	3	$R_2' = \frac{R_2}{3}$
w	1	0	0	0	0	0	-1	0	$R_w = R_w + R_2$
w	1	3	2	-4	0	-1	0	3	$R_w = R_w - 3R_2'$
x_4	0	0	$-1/3$	$\boxed{23/3}$	1	$2/3$	$-2/3$	4	$R_1' = \frac{3}{23}R_1$
x_1	0	1	$2/3$	$-4/3$	0	$-1/3$	$1/3$	1	$R_2 = R_2 + \frac{4}{3}R_1'$
w	1	0	0	0	0	0	-1	0	
z	1	-1	1	-3	0	0	0	0	$R_3 = R_3 + R_2$
z	1	0	$5/3$	$-13/3$	0	$-1/3$	$1/3$	1	$R_3 = R_3 + \frac{13}{3}R_1$
x_3	0	0	$-1/23$	1	$3/23$	$2/23$	$-2/23$	$12/23$.	
x_1	0	1	$14/23$	0	$4/23$	$-17/69$	$17/69$.	$117/69$.	
z	1	0	$34/23$	0	$13/23$	$1/23$	$-1/23$	$75/23$.	

The optimal solution to the primal is $(x_1^*, x_2^*, x_3^*) = (\frac{117}{69}, 0, \frac{12}{23})$ $z^* = \frac{75}{23}$

(iii) The optimal solution of the dual is $(y_1, y_2) = (\frac{13}{23}, \frac{1}{23})$ (see N.B. above)! $y^* = \frac{75}{23}$.

$2(\frac{13}{23}) - 3(\frac{1}{23}) = 1$, $\frac{13}{23} - 2(\frac{1}{23}) = \frac{11}{23} \geq -1$, $5(\frac{13}{23}) + 4(\frac{1}{23}) = 3$. $\therefore$ solution is feasible