1/ (a) \[ D \text{ is minimize } 2y_1 + 4y_2 \]
\[ s.t. \begin{align*}
2y_1 + y_2 &\geq -1 \\
y_1 + 2y_2 &\geq 2 \\
3y_1 + 2y_2 &\geq -1 \\
y_1, y_2 &\geq 0
\end{align*} \]

(b). The first primal constraint is satisfied with equality \( y_2^* = 0 \).
\( x_2 > 0 \) \Rightarrow \text{ The second dual constraint is satisfied with equality } \Rightarrow y_1^* = 2.

The optimal solution at 0 is \((y_1^*, y_2^*) = (0, 2)\) with \( w = 2 \).

2/ \[ D \text{ is minimize } 4y_1 + 3y_2 + 5y_3 + y_4 \]
\[ s.t. \begin{align*}
y_1 + 4y_2 + 2y_3 + 3y_4 &\geq 7 \\
3y_1 + 2y_2 + 4y_3 + y_4 &\geq 6 \\
5y_1 - 2y_2 + 2y_3 + 3y_4 &\geq 5 \\
-2y_1 + y_2 - 2y_3 - y_4 &\geq -2 \\
y_1 + y_2 + 5y_3 - 2y_4 &\geq 3 \\
y_1, y_2, y_3, y_4 &\geq 0
\end{align*} \]

with \( x^* \), the third constraint is satisfied with inequality \( y_1^* = 3 \).

The second, third and fourth primal variables are positive \( \Rightarrow \bar{y}_1, \bar{y}_2, \bar{y}_3, \bar{y}_4 \geq 0 \).

The corresponding dual constraints must be satisfied with equality \( \Rightarrow \bar{y}_1^* + \bar{y}_2^* + \bar{y}_3^* = 6 \)
\( 5\bar{y}_1^* - 2\bar{y}_2^* + 2\bar{y}_4^* = 5 \)
\( -2\bar{y}_1^* + \bar{y}_2^* - \bar{y}_4^* = -2 \).

Solution is \((y_1^*, y_2^*, y_3^*) = (1, 1, 1)\) but not \( y_4^* \) so is not possible.

The primal and dual solutions are optimal. Solution is not optimal.