TUTE II: Travelling Salesman Problem (using DP)

- A salesman has to travel between n cities \( \{1, 2, ..., n\} \) with the direct distance between them given by \( d(i, j) = \text{distance from city } i \text{ to city } j \) \( \forall i, j \in \{1, 2, ..., n\}, \ i \neq j \)

- Determine the shortest route that visits each city exactly once, except the home city where the tour starts and finishes. We generally assume (without loss of generality) that city 1 is the home city.

- Using DP we deconstruct the main tour into subtours, which are easier to solve.

KEY IDEA:

Suppose we are at city i and still have to visit the cities in the set \( S \)
\((i, S)\)

Suppose we decide to go to city j next \( \rightarrow \) then we travel the distance \( d(i,j) \)

We are now at city j and still have to visit the cities in the set \( S \setminus \{j\} \)
\((j, S \setminus \{j\})\)

TSP: DP functional equation

- Let \( f(i, S) \) be the shortest subtour given that we are currently at city i and still have to visit the cities in the set \( S \) before we return to the home city (City 1)

- Then \( f(i, \emptyset) = d(i, 1) \) (starting point of shortest tour \( \rightarrow \) no cities left to visit \( \Rightarrow \) go straight home)

\( f(i, S) = \min_{j \in S} \{d(i,j) + f(j, S \setminus \{j\})\}, \ S \neq \emptyset \)

where \( S \setminus \{j\} = \{k \in S : k \neq j\} \).

- The original problem is then given by \( f(1, C) \) where \( C = \{2, 3, ..., n\} \). (\( \therefore \rightleftharpoons = f(1, C) \))

PROBLEM 2, 2001 EXAM

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & - & 2 & 4 & 3 \\
2 & 3 & - & 2 & 4 \\
3 & 4 & 4 & - & 1 \\
4 & - & 5 & 3 & 1 \\
\end{bmatrix}
\]