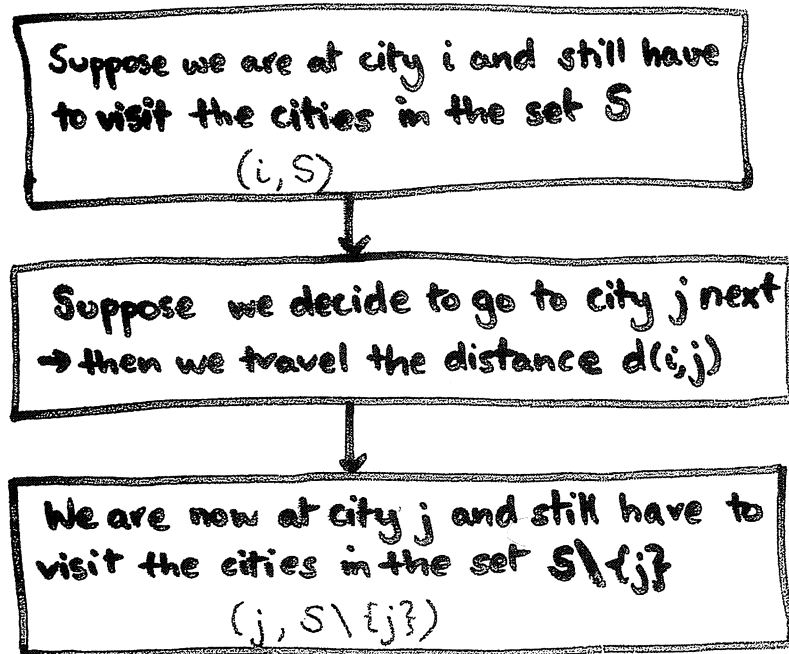


## TUTE 11: Travelling Salesman Problem (using DP)

- A salesman has to travel between  $n$  cities  $\{1, 2, \dots, n\}$  with the direct distance between them given by  $d(i, j) = \text{distance from city } i \text{ to city } j \forall i, j \in \{1, 2, \dots, n\}, i \neq j$
- Determine the shortest route that visits each city exactly once, except the home city where the tour starts and finishes. We generally assume (without loss of generality) that city 1 is the home city
- Using DP we deconstruct the main tour into subtours, which are easier to solve.

### KEY IDEA:



## TSP: DP functional equation

- Let  $f(i, S)$  be the shortest subtour given that we are currently at city  $i$  and still have to visit the cities in the set  $S$  before we return to the home city (City 1)

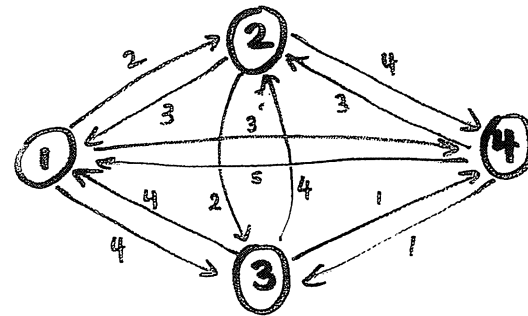
- Then  $f(i, \emptyset) = d(i, 1)$  (starting point of final eqn  
→ no cities left to visit  
ie go straight home)

$$f(i, S) = \min_{j \in S} \{d(i, j) + f(j, S \setminus \{j\})\}, \quad S \neq \emptyset$$

where  $S \setminus \{j\} = \{k \in S : k \neq j\}$ .

- The original problem is then given by  $f(1, C)$  where  $C = \{2, 3, \dots, n\}$ . ( $\therefore z^* = f(1, C)$ )

### PROBLEM 2, 2001 EXAM



1	2	3	4
1	-	2	4
2	3	-	2
3	4	4	-
4	5	3	1