620-261 Introduction to Operations Research

ASSIGNMENT 4

Post in boxes by 3.00 pm on Monday 14th April 2008

1. In Assignment 2, I asked you to solve a three-dimensional problem. This involved formulating it as a linear program and then finding a way to solve it. I did not specify how you should do the latter - I was interested in what you would come up with. In this question we return to the problem. The tasks are
   (a) Solve the problem using the simplex algorithm.
   (b) Write a couple of sentences to explain the advantages of the simplex algorithm over the method of solution that you used last time.

   NB: A couple of you used the simplex algorithm last time. Others did not attempt to solve the problem. If you are in either of these categories, in part (b) above, you should compare the simplex algorithm with the method that involves enumerating all basic solutions, checking each for feasibility and then evaluating the objective function at each basic feasible solution.

   The formulated problem was

$$\text{Maximise} \quad z = 12x_1 + 6x_2 + 4x_3$$

   subject to

$$\begin{aligned} 0.9x_1 + 0.3x_2 + 0.1x_3 &\leq 3,000 \\ 0.1x_1 + 0.5x_2 + 0.3x_3 &\leq 5,000 \\ 0.2x_2 + 0.6x_3 &\leq 10,000 \end{aligned}$$

   with x_1, x_2 and x_3 non-negative.

2. Show that the simplex algorithm does not find an optimal solution to the LP

$$\text{Maximise} \quad z = 2x_1 + x_2$$

   subject to

$$\begin{aligned} x_1 - 2x_2 &\leq 2 \\ -x_1 + x_2 &\leq 1 \end{aligned}$$

   with x_1 and x_2 non-negative. Draw a graph that explains your result.

   Please turn over for Question 3.

3. Solve the following LP using the simplex algorithm

$$\text{Maximise} \quad z = x_1 + x_2$$

subject to

$$\begin{aligned} x_1 - x_2 &\leq 1 \\ x_1 + x_2 &\leq 2 \\ -2x_1 + x_2 &\leq 0 \end{aligned}$$

with x_1 and x_2 non-negative.

In the final tableau, select a non-basic column ℓ with $c_\ell = 0$, and perform one more iteration of the simplex algorithm with this column as the pivot column. Is the new solution feasible? Is it optimal? Draw a graph to explain what is happening.