1. Write the dual of the following linear programming problem by first writing it in standard form.

\[
\begin{align*}
\text{max} & \quad 3x_1 + 2x_2 - x_4 \\
\text{subj to} & \quad 4x_1 + 3x_2 + x_3 = 10 \\
& \quad 5x_1 - 6x_2 + x_3 \leq 12 \\
& \quad 6x_1 + 7x_2 \geq 11 \\
& \quad x_1 \geq 0, \ x_3 \geq 0, \ x_4 \geq 0, \text{ w.r.s}
\end{align*}
\]

2. Consider the linear program, \( P \), given by

Maximise \( z = x_1 + 4x_2 + 3x_3 \)

subject to

\[
\begin{align*}
x_1 + 5x_2 + 4x_3 & \leq 14 \\
3x_1 + 2x_2 + x_3 & \leq 4 \\
\end{align*}
\]

with \( x_1, x_2 \) and \( x_3 \) non-negative.

(i) Write down \( D \), the dual of \( P \).

(ii) Solve \( P \) using the simplex algorithm.

(iii) Write down the optimal solution of \( D \).

3. Consider the linear program, \( P \), given by

Maximise \( z = x_1 - x_2 + 3x_3 \)

subject to

\[
\begin{align*}
2x_1 + x_2 + 5x_3 & \leq 6 \\
-3x_1 - 2x_2 + 4x_3 & \leq -3 \\
\end{align*}
\]

with \( x_1, x_2 \) and \( x_3 \) non-negative.

(i) Write down \( D \), the dual of \( P \).

(ii) Solve \( P \) using the simplex algorithm.

(iii) By observing the optimal tableau of \( P \), write down the optimal solution of \( D \). Verify that this is a feasible solution for \( D \).